ActivMathComp – Computers and Active Learning as support of a whole learning environment to Calculus / Mathematical Analysis

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Abstract

Research reports many success cases where computers and active learning enhance students understanding. *ActivMathComp* is a whole learning environment created to this research to teach Calculus/Mathematical Analysis 1 (MA1) to Civil Engineering students of the Polytechnic Institute of Lisbon, Portugal. Some principles guide the whole learning environment: Students are active and collaborate with colleagues during classes; Computers are embedded in all activities, as a communication, interaction, and computational tool; Students use a set of digital interactive learning documents created to the approach that include the slides, the daily diary, the exercises list, the theory book, etc; Students answer weekly to short quizzes with immediate feedback on a Learning Management System; The teacher/student relationship is grounded on trust, on mutual understanding and on students' involvement on their own learning; Students explore concepts from multiple perspectives (visual, verbal, numerical, and analytical); Students keep in touch with mathematical applications, particularly in Civil Engineering.

ActivMathComp was applied to a class with the students that choose to subscribe it; they could not be randomly assigned because they must take their personal laptop to every class. This way, it was created a quasi-experiment to answer the research question: Does *ActivMathComp* approach generate higher grades and higher success rate than the traditional approach to teach MA1 course? A questionnaire was applied to all MA1 students to study if the students that participated in *ActivMathComp* were different from the others in terms of background, attitudes or behaviours. A second questionnaire and two focus groups were applied to *ActivMathComp* students in order to understand what evaluation they have made of the approach. Participating students got significantly higher grades and higher success rate than other students (some threats to validity remain). Participating students evaluated the learning environment as highly positive in nearly all aspects.

Introduction

Mathematics has high failure rates from earlier grades to higher education (Machado, 2006; Jackson, Johnson, & Blanksby, 2014; Bardini & Pierce, 2015; Wandel, Robinson, Abdulla, Dalby, Frederiks, & Galligan, 2015; Wilkes & Burton, 2015). Particularly, Mathematical Analysis/Calculus 1 (MA1) arises as a course with worrying failure rates, as is stated in several studies (e.g. Treisman, 1992; Mumford, 1997; Husch, 2002; Domingos, 2003). For example, the MA1 failure rate at the Instituto Superior de Engenharia de Lisboa (ISEL), in 2006/07, was of 89%; in the USA, in 1987, from the 600 000 students taking college calculus, only 46% obtained grade D or above (Anderson & Loftsgaarden, 1987). Moreover, students usually have low understanding of concepts, even the approved students (Domingos, 2003). Students 'frequently come to view calculus as strictly procedural' (Zerr

2009, p.16), tending to memorize processes instead of deeply understanding them (Domingos, 2003). Tall (1993) emphasizes the difficulties that students have in specific concepts, such as limits and processes involving infinity.

The identification of these problems suggested the need for change. In addition, 'A general dissatisfaction with the calculus course has emerged in various countries round the world', reported, from England, Tall (1993, p.1). Artigue, Authier, Bessot, Delale, Germain, Jarraud, Lanier, Le Goff, Legrand, Robert, Robinet, Rogalsky, Sacre (1990) belonging to Institut de Recherche sur l'Enseignement des Mathématiques in France have pursued the need to make the subject development more meaningful to students. For example, in USA, the Calculus Reform movement involved several national institutions and many universities. Under this reform, Gleason and Hughes-Hallet (1992, p.1) professors at Harvard University, wrote that they 'believe that the calculus curriculum needs to be completely rethought'.

Technology Enhanced Learning (TEL) has become ubiquitous - Information and Communication Technologies (ICT), Computer Algebra Sysytems (CAS); Learning Management Systems (LMS), Computer-Aided Assessment (CAA), among many others have opened a new world of possibilities to enhance learning. Recommendations to take advantage of TEL to deepen students learning come from reputable institutions like NCTM (2000); from researchers at the beginning of technology boom (Kaput, 1994); and from current researchers (Lawson, 2002; Teodoro, 2002; Blackwell, Trzesniewski, & Dweck, 2007; Rylands & Shearman, 2015; Hill, Sharma, & Johnston, 2015). 'The new teacher will become a facilitator that adopts a technology, either in the classroom or online, and promotes learning using it' (Caprotti, Seppala, & Xambó, 2007, p. 299).

Many problems may be addressed by the use of TEL in all its potentialities (Machado, 2006; Caprotti, Seppala, & Xambó, 2007; Yeung, Schmid, & Tasker, 2008; Sujarittham, Emarat, Arayathanitkul, Sharma, Johnston, & Tanamatayarat, 2016). TEL allows to address difficulties associated with high student diversity. It allows to educators to tailor subject content and design learning environments according to student characteristics: learning pace, performance, individual processes, cognitive style, background, culture, etc.

To first-year Calculus subjects, there is a wide range of software available and it is used in diverse innovative ways. Tall, Smith and Piez (2008, p. 207) believe that Calculus is the area of mathematics that gets the 'most interest and investment in the use of technology'. For example, Borchelt (2007) believes that the use of Computer Algebra Systems (CAS) has become unavoidable in College mathematics. Ersoy and Akbulut (2014) studied approaches using of CAS, Wolfram Alpha and linear web pages. However, there is (still) a delay in the use of technology in mathematics: the difficulty of writing mathematical symbols using a computer (Caprotti, Seppala, & Xambó, 2007).

On the other hand, on a didactic approach, in opposition to traditional lectures, Active Learning is well established. Active Learning may signify many different approaches but all have the property that students are not passive in classes. According to Bonwell and Eison (1991) to achieve active learning students must do more than just listen: they must read, write, discuss, and be engaged in problem solving; and they also should practice higher order tasks such as analysis, synthesis, and evaluation.

Prince (2004) in his paper 'Does Active Learning work? A review of research' examines different kinds of active learning, from the introduction of student activity into the traditional lecture, to the introduction of activities promoting student engagement, or collaborative or cooperative learning or even problem-based learning. He found positive effects for all studied forms of active learning.

Collaboration is one expression of active learning. Johnson, Johnson and Smith (1991) made a review of more than 90 years of research and found that cooperation improved learning relative to individual work.

Nowadays, active learning is used, with success, in several approaches with different kinds of students, both in top universities and in other higher education institutions. Examples of this include: TEAL (Dori & Belcher, 2004) in Massachusetts Institute of Technology (MIT); SCALE-UP (Beichner, Saul, Abbott, Morse, Deardorff, Allain, Bonham, Dancy & Risley, 2007) in North Carolina State University; Peer Teaching (Lasry, Mazur, & Watkins, 2008) in Harvard University; Interactive Lecture Demonstrations (Georgiou & Sharma, 2015) in The University of Sydney; ASELL: Advancing Science by Enhancing Learning in the Laboratory (Yeung, Pyke, Sharma, Barrie, Buntine, Da Silva, Kable, & Lim, 2011) in University of Adelaide.

To this study we intended to create a learning environment to teach theoretical-practical classes of Calculus/ Mathematical Analysis 1 (MA1) that could generate higher grades and higher success rate as a consequence of a deep understanding of concepts by students. The fundamental principles of the learning environment, which we named as *ActivMathComp*, are:

- Students are active and collaborate with colleagues during classes;
- Computers are embedded in all activities, for communication, interaction, and computational tools;
- Students use digital Interactive Learning Documents (ILDs): a set of interactive documents to support learning that include the slides, the daily diary, the exercises list, the theory book, etc.
- Students explore concepts from multiple perspectives (visual, verbal, numerical, and analytical);
- Students keep in touch with mathematical applications, particularly in engineering;
- Students answer to frequent short quizzes with immediate feedback on a Learning Management System;
- Teacher/student relationship is grounded on trust, on mutual understanding and on students' involvement on their own learning.

The *ActivMathComp* does not follow only one approach mentioned in literature. It integrates different aspects of several approaches. It is complete in the sense that it gives directions for the learning design, teaching method, the type of approach to the curriculum, the provision of support materials, the guidance provided to students for assessment tasks and self-assessment, the relationships among students/teachers/colleagues, the use of computers as: communication tools, computational tools, and as tools to explore concepts.

The overall purpose of this study is, besides the design of a new learning environment, to assess the outcomes of the implementation of *ActivMathComp* in a course. The main issue to investigate is whether *ActivMathComp* improves students' performance, both in higher success rate and in improved grades. It was also addressed students' vision about the whole approach; about the Interative Learning Documents (ILDs); and about the weekly online quizzes on *Moodle*.

Method: *ActivMathComp* design and principles

In this research, a group of students (a class) was subject to an intervention and we study if those students had different outcomes from the control group, the others students. The intervention was a whole learning environment that we call *ActivMathComp*. This section describes its design according to the subjacent principles. First section provides the context to which it was applied. Next is explained how *ActivMathComp* fostered active and collaborative learning. Followed by the principles subjacent to the design of the Interactive Learning Documents (ILDs). Finally is described how the computer integrates the learning environment.

Context of *ActivMathComp*

The participants of this study were the 519 subscribed students of the regular course of Mathematical Analysis/Calculus 1 (MA1) at Instituto Superior de Engenharia de Lisboa (ISEL) of Polytechnic Institute of Lisbon, Portugal. The course belongs to the first semester of Civil Engineering graduation. The number of “ghost students” who were subscribed but do not effectively try to get success was huge, only 295 students that came to, at least, one assessment were taken into account. MA1 was taught in eight classes, including the intervention one, by four professors. The medium attendance by class, during the semester, was around 20 students.

ActivMathComp was implemented in the first semester of 2009-10 in a class with students not randomly chosen since they were required to take their personal laptop to every class. It was made public that *ActivMathComp* was available in MA1 to all enrolled students and 16 choose to enrol the class. This low number of students enrolled was, probably, due to the awkward timetable.

One could think that students that have a laptop were more financially advantaged students, meaning a greater access to outside resources, and/or more motivated for success but, in Portugal, due to governmental policies in the ‘technological plan’, in 2009, at the end of high school nearly all students have a personal laptop. Hence, very few students were excluded from participating by that reason. (Three students usually did not take their laptop to classes but they make the activities and the mini-tests anyway since teacher used the data-show to show the ‘questions’ and they answered on paper.)

MA1 lasts a semester and its classes are theoretical-practical which means that theory and practice are taught together in every class. Usually there are four classes every week lasting 1h30 each, and it was what happened to all the regular classes of MA1, only the *ActivMathComp* class had two classes of 3h a week to do not make its timetable even worse for students.

The syllabus of MA1 is Logic, Set Theory, Induction Method, Real numbers, Sequences, Numerical series, Topology, Functions, Differentiability, Power series, Antiderivatives, Integrals and Improper Integrals.

The *ActivMathComp* students were assessed by the course normal assessment, i.e., students get approval if they obtain a grade higher than 9.5 values (out of 20 values) in: the average of two tests that took place throughout semester; or in the first final exam; or in the second final exam. In this class, in addition to the course normal assessment, students had taken 14 quizzes in Moodle (nearly one per week) that were answered during classes, usually at the end, and lasted about 20 minutes. In that moment students were spread throughout the classroom and the teacher made a strong vigilance of those students, since they had internet connection, to ensure that they do not communicate with each other; she usually was seated at the back of classroom where she was able to see all laptop screens. Parameters in quizzes were not randomized since it demanded much more work to conceive the quizzes and it was not worthwhile for such a small amount of students.

To *ActivMathComp* students, at the final normal grade was added a grade between 0 and 2 values, only when the normal grade was higher than 9.5 values, proportional to the average of the 10 best grades in quizzes. These extra values pretended to motivate students to engage in quizzes and were a reward for the extra work made by the students who decided to make the quizzes, compared to the others students. These values were not taken into account in data analysis. Quizzes questions were of closed-response (multiple choice, numeric answer, short answer, etc.) and students got the grade in the first time they submit an answer. However, students could do more attempts that allowed them to re-try the quiz and practice until they arrive to the correct answer.

The ILDs materials of *ActivMathComp* were available online on Moodle page to all MA1 students, and the teacher used it, not totally but most part, also in her another class. The others three teachers did not use it. The quizzes were only available to *ActivMathComp* students. The "traditional" material of the course: a book made by the MA1 responsible teacher and a list of proposed exercises was available to all students.

Active learning and collaborative work

Active student-centred learning predominated in *ActivMathComp* classes. Most of the time students were effectively working and the teacher only intervened when a student requested support. Students worked collaboratively and autonomously. The classroom atmosphere was supportive and empathetic, neither showing harsh criticism nor indifference.

A typical *ActivMathComp* class began with the teacher explaining an issue for about five minutes and students exploring it during the remaining 85 minutes. A concept explanation usually began with a presentation of a real-life problem. This was followed by its resolution and its generalization to the corresponding concept. Students then thought about the concept properties answering interactive questions about it (see example in Figure 1) in ILDs, on their personal laptop.

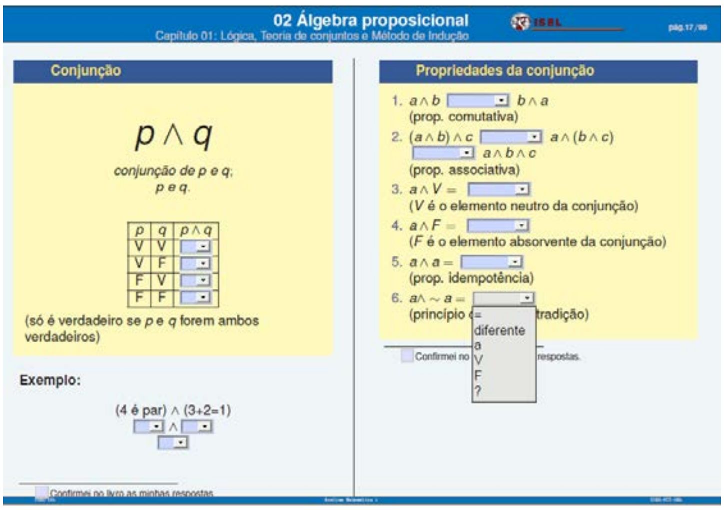

Figure 1: Combo boxes to promote interaction (V means True; F means False) (All figures are translated on Appendix A).

Next, the teacher presented a list of increasing difficulty exercises (ensuring they were in the learners zone of proximal development) and told students: ‘Now it’s up to you! Solve the exercises and when you have doubts ask your fellow students or me. A strategy to get students actively working was to set a range of exercises to ensure that all students were engaged in the process. Students know that the teacher will only solve the exercises at the class end, in a very quick and brief way. Thus, during the rest of the class, students try to solve the exercises by themselves.

When teacher realized that a student was not working, she asked him: ‘How can I help you?’, ‘Are you lost in the subject? Did you miss some classes? May I help? ...’ and usually student accepted and learned to do what was required. When students had questions they asked their peers and, when nobody in the neighbourhood knew the answer, they called the teacher. Usually the teacher did not solve the problem but gave them ideas to help to reach the solution by themselves. As each student worked independently and at their own pace, and students had different rhythms, at the end, each student had completed a different number of exercises, some had reached complex exercises, and others had done only the simplest, but all had solved the most basic exercises by themselves, and could master, in fact, the basics of the concept. Students with more difficulties perceived that they actually had managed to understand these basic exercises, and with just a little more work they would manage to understand the others. Students that succeeded in easier questions went to the last, more challenging exercise where they were able to exercise high-level skills.

Interactive Learning Documents (ILDs)

This approach created a set of interactive documents as learning support. It was a basis to students’ work: the slides, the daily diary, the exercises list, the theory book, etc. Their conception took into account a set principles described below.

Interaction

These documents promote active learning, since they are not static. They are not as traditional books/materials that just exhibit concepts, exercises and the resolution of exercises. Using ComboBoxes, CheckBoxes, TextFields, etc. it forces students to interact, to distinguish what does and what does not make sense for each concept. For example, instead of the teacher just presenting the properties of integrals and students nod their heads affirmatively, ILDs make students to complete those properties (see Figure 2).

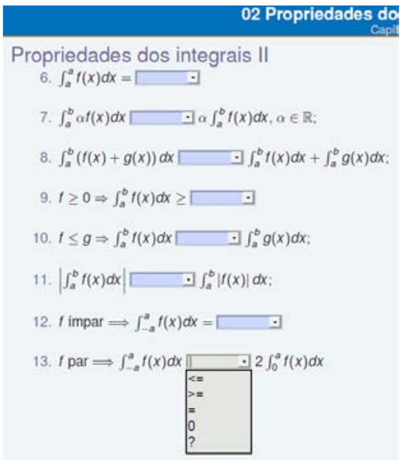

Figure 2: Interaction to promote Active Learning.

Application problems

The ILDs include Mathematics applications to many subjects. Since ILDs were made to Civil Engineering students, we preferred, when available, to include applications to Civil Engineering (see Figure 3). Applications were made to motivate students (to make students avoid the idea that 'mathematics is not good for anything ... has no interest'), to enable students to understand deeply the concepts, to develop skills for solving application problems and so, to work the higher levels of Bloom's educational objectives taxonomy: apply, analyse, evaluate and create. Since the course general assessment does not include any type of 'real life' applications it was not fair for students to spend much of the short time available to teach how to explore applications. Therefore, the strategy was to explore concepts applications only for the motivation purpose, to learn a given concept and to make exercises that allow a better understanding of the concept.

Figure 3: Application problem (Suppose that you want to make a cylindrical tank to contain 1000m^3 of water, using the least material possible. Suppose that the material of the top and the bottom is the same that the wall material. Which are the tank best dimensions?).

Multiple representations

As more representations of the same concept are used, the better it is. It allows students to create links between different representations and achieve a deeper understanding. According to Calculus Consortium based at Harvard University, whenever it makes sense, concepts should be presented in an analytical, visual, numerical and verbal way (an example in Figure 4). Furthermore people learn in different ways, have different learning preferences: some are more verbal, others more visual; some more global, others more sequential; some more active, others more reflective; etc. and the more learning preferences we use, the more students we reach.

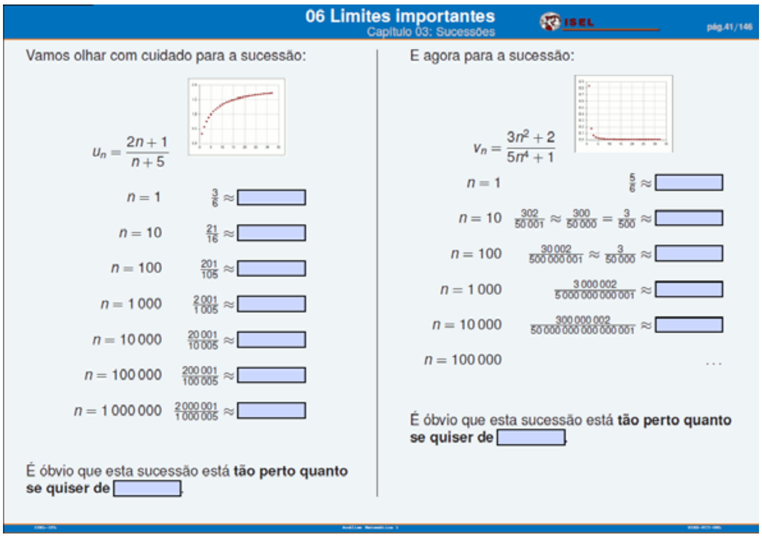

Figure 4: Limits presented in an analytical, visual, numerical and verbal way.

From particular cases to abstract generalization

Is easier to a student to understand a particular case and then generalize it, than the reverse, as it is traditionally made in mathematics, see Figure 5.

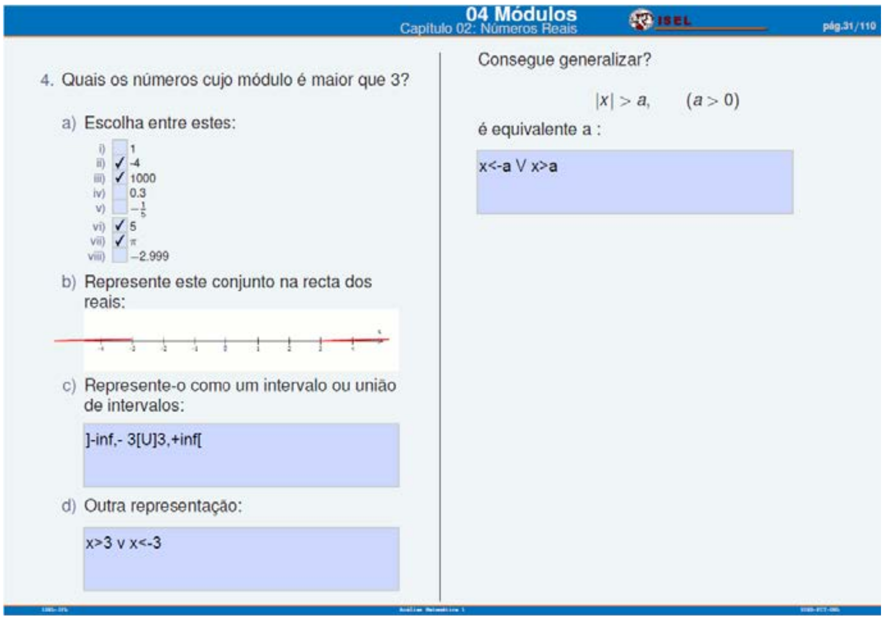

Figure 5: From particular to general. Studying $|x| > 3$ to get to $|x| > a$.

Proximal Development Zone

According to Vygotsky's theory (1980), *ActivMathComp* exercises respect students Proximal Development Zone. Teacher attempts to anticipate students' problems and promote adequate theme scaffolding. Exercises sequence has little increasing difficulty to allow the student to progress from one to the next by himself or with a little help from a colleague or from the teacher. For example, if, in the following example (Figure 6), students faced only a few questions, it would be much more difficult for them to evolve from one exercise to another, and understand this concept logic.

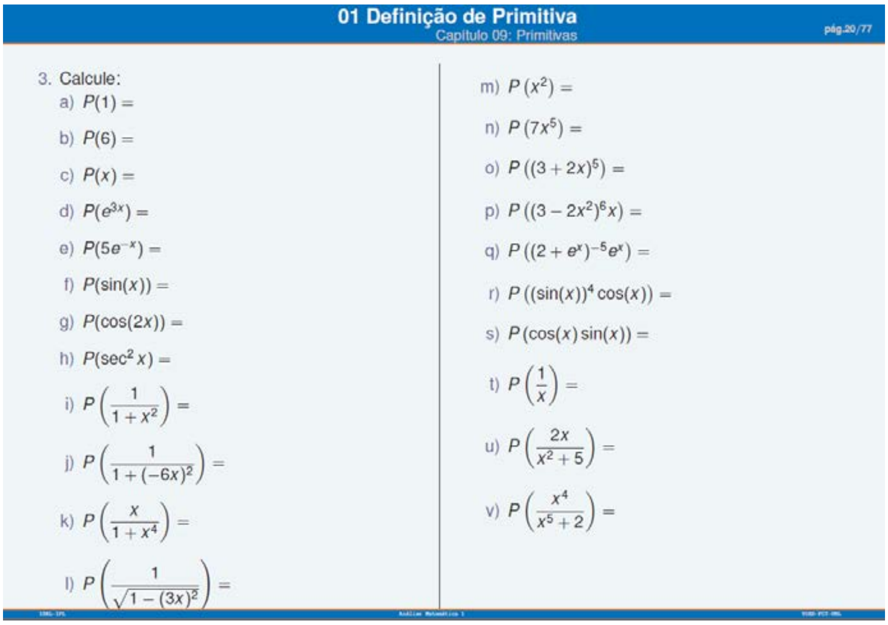

Figure 6: Exercises respecting students Proximal Development Zone to learn antiderivatives.

An objective at a time

In *ActivMathComp* when we want that student learn a concept we do not mix it with another (at least in the beginning, later on we may relate concepts, or work several concepts at once, but not in the begining).

Appellative

Finally, considering the aesthetic qualities, why not give a light, pleasant, modern, and dynamic look to Mathematics? (See Figure 7) To this purpose, we integrated colourful and dynamic photos; had chosen a not very formal font to write; looked for an agreeable format style; introduced modernity and dynamism using ComboBoxes, CheckBoxes and TextFields.

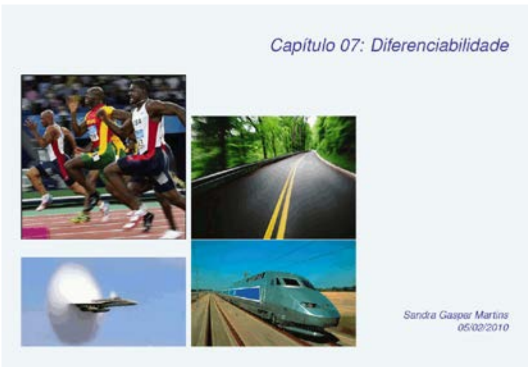

Figure 7: An appellative look.

Computer

The teacher and students took laptop daily to classes to use: the ILDs, the quizzes, software, files, links, etc. A *Moodle* web page gave support to *ActivMathComp* containing all the materials, links, quizzes, forum, wiki, honour board, etc.

Software

A problem arose about the effective use of software in *ActivMathComp*. In MA1 course, during assessment, students cannot use software of any kind (neither computers nor calculators). If teacher make students spend much time in class to learn how to work with several computer programs it would be adverse affecting *ActivMathComp* students since they would have less time to work all the other. Hence the use of software was developed in order to show to students files previously made by the teacher and not from the perspective of being the students themselves to create the files.

Those files were created in *Excel*, *Modellus* and *WxMaxima*. Those files allow concepts exploration (for example in numeric and graphical form) or show how some concepts allow to model the reality, or showing models of concepts, or showing animations to illustrate concepts, etc. Files were then used in the perspective of being one more element that could support concepts deep understanding, since it is richer in concepts representations and has a positive influence on students' motivation.

The software/files used were:

Modellus – to modelling mathematical entities, to explore concepts and connections between concepts, to create and explore physical phenomena and the underlying mathematical relationships behind the phenomena. For example: Using a photo, the teacher modelled the Vasco da Gama Commercial Centre roof and a jet of water using parabolas. She used an animation of the trigonometric circle to introduce the sinus function, associating it with its values in a table and its plot. She used Modellus to get an estimate of the area of a Batalha Monastery window to introduce Riemann sums, i.e. to introduce integrals. And she used animations to develop intuition about the concepts of sequence, monotonous sequence, etc.

Excel (the Microsoft Office Spreadsheet) – mainly to explore numerically and graphically the concepts. Namely, explore the definition (of Cauchy) of a sequence limit using particular values of □. In a simila
series, relating it with its sum and also used the spreadsheet to get estimations of the series sum. In both cases, the goal was to materialize concepts and to develop intuition. Moreover it was shown, intuitively, that with a spreadsheet we cannot verify if a property is valid to all natural numbers but we may do that by induction. With graphics and tables the concept of sequence and its properties become more intuitive.

WxMaxima and *Wolfram-Alpha* (both Computer Algebra Systems) – to make symbolic calculations and graphics. For example, to validate the result after making the analytical calculation of limits of functions and sequences, series sum (and their convergence), derivatives, integrals, etc. It was also used to plot functions and then to visualize where are their maxima, minimum, asymptotes, monotony, etc. and to explore functions composition. Some *ActivMathComp* students used CAS, in class and in quizzes, by their own initiative, validating the results gathered analytically.

Adobe Reader – to read the ILDs (in PDF).

Mathtype – a professional mathematics editor for Microsoft Word, it was used to write mathematics on ILDs.

Quizzes with immediate feedback

Quizzes lead students to realize the performance level that they achieved and allowed them to verify if the performance was the intended one or not. Often students are unaware of what they do not know. The sooner we confront them with the level that they, in fact, reached, the more chance they have to change their behaviour and the amount of time spent studying the course, if needed. We decided to assign a grade to quizzes because students give more value to work directly valued in grades.

Applets

The animation achieved with Applets (or similar tools) allowed the class to explore or to get a visual idea of concepts and its properties.

Results

The participants of this study were the 519 students of the regular course of MA1 at Instituto Superior de Engenharia de Lisboa (ISEL) of Polytechnic Institute of Lisbon including the class of 16 students that participated in the intervention. The 295 students that came, at least, to one assessment were taken into account.

This research main question is: RQ1 - Does *ActivMathComp* approach generate higher grades and a higher success rate than the traditional approach to teach the Mathematical Analysis 1 (MA1) course? Were also addressed the following secondary research questions: RQ2 - What was students' vision about the whole *ActivMathComp*? RQ3 - And about the Interative Learning Documents (ILDs)? RQ4 - And about the weekly online quizzes?

The *ActivMathComp* evaluation was made using a quasi-experiment, not an experiment since students could not be randomly assigned to class because they must take a personal laptop to class. We used the following tools: A questionnaire was applied to the MA1 students (both participants and non-participants) to study if the students that followed *ActivMathComp* were different from the others in terms of background, attitudes or behaviours. Two focus groups allowed a closer approach to the *ActivMathComp* students and give support to the construction of a second questionnaire that was given to *ActivMathComp* students in order to understand what evaluation they have made of the approach, including the assessment of ILDs and quizzes. Grades of all students were also collected to find if there were significant grade differences among participating students and the others.

Research Question 1: Grades and Success Rate

ActivMathComp success rate almost doubled the others students success rate (see Table 1).

Table 1: Quantity of assessed and approved students and success rate, by groups.

	Subscribed	Assessed	Approved	Approved/Assessed
ActivMathComp	16	14	9	64.3%
Others students	503	280	92	32.9%

Table 2 shows that the average grade of participants (8.8 values) is much higher than the average grade of the formal comparison group (5.7 values). The maximum grade of all students also belongs to a participant of the study (16.5 values).

Table 2: Comparison of MA1 grades between participants and the other students.

	N	Aver.	Std. Dev.	Std. Error	Min	Max
Participants	14	8.8	4.8	1.3	0.7	16.5
Others students	281	5.7	4.1	0.2	0.0	15.5

An ANOVA test shows that grades differ significantly from one group to the other, $F(1,293)=7.35$, $p<0.05$.

As the normality of the variable is not warranted (Kolmogorov-Smirnova and Shapiro-Wilk) and the number of participants is very small and much less than the number of the 'others students', ANOVA may not be completely robust. A non-parametric test, the Independent-Samples Mann-Whitney U test, was also performed and the result of $p=0.014$ leads us also to reject the null hypothesis which states that 'grades distribution is the same across groups'.

Therefore, it may be concluded that participants grades are significantly higher than grades of the others students of MA1.

A natural question arises: Since *ActivMathComp* students were not arbitrarily assigned, are they a group with different characteristics from the general students? Since there was no official data about students' characteristics, a questionnaire was applied to all students. The conclusions were that there is no big difference between the participants and the comparison group in terms of personal characteristics, background, attitudes and behaviours.

Research Question 2: Students' vision about *ActivMathComp*

To address students' perception of the interest/usefulness of the whole learning environment of *ActivMathComp* it was made two focus groups and a questionnaire to the participating students.

In questionnaire, participating students indicated their degree of agreement with the following statements:

- I liked the way lessons were conducted.
- It seemed useful the fact of not spending time copying the theory (which was on the slides) and using it to solve exercises.
- It was important to see not only the analytical part of mathematics but also the graphical and numerical.
- It was interesting to see some applications of themes that were studied.
- The fact that the subject was shown interactively and not as a presentation was useful. For example, the teacher instead of saying that the properties are A < B and C > D, asks students to fill with inequalities the relations between A and B and between C and D.

Table 3. Number of participants agreeing with the statements about *ActivMathComp*

	N.O.	Nothing			Medium			A lot		
	0	1	2	3	4	5	6	7	*N*	Average
Liked lessons							9	6	15	6.4
Useful not copy theory					1	2	5	7	15	6.2
Imp. Graphic/Numeric					1	4	6	4	15	5.9
Interesting Applications					2	3	7	3	15	5.7
Useful Interactivity				1	3	5	6		15	5.1

From table 3 we may conclude that students really liked lessons including the particular issues like not to waste time copying theory, the graphical and numerical view beyond the analytical part, the use of Mathematics applications and also found useful the interactivity.

Given the two teaching methods: **Teaching Method 1 (TM1):** the teacher presents the themes for a very short time and the remaining time circulates among students helping them to solve the exercises (the students interact with each other in order to help each other). And **Teaching Method 2 (TM2)**: most of the time the teacher explains the theme and solves the exercises on the blackboard asking questions to the students in general. TM1 was the teaching method adopted in *ActivMathComp* and TM2 was the teaching method adopted in the other, traditional, classes. In the answers to questionnaire 12 out of 13 students state that prefer TM1 and 2 did not answer.

Concerning the question if they ”would return to *ActivMathComp*” (if they need to make the course again) the 12 respondents answered “yes”, 1 answered “no opinion” and two did not answered.

Answering the question: “From all that was addressed before, what had the biggest impact in your success/failure?” the following topics were the most mentioned: Quizzes (5); Few students (5); Material to keep up with lesson (4); Relationship between fellow students (3); Clear and slow explanation of the subject (2); Relationship between teacher and students (2); Availability of teacher (2); Good Timetable (2); Many practice (2); Learning Methodology (2).

Students felt that the fact of being few students was important. The teacher does not agree with that statement since she applied that environment to classes with 50 students and can reach all the students (since students collaborate with each other and rarely need teacher) and feels the same feedback.

From focus groups and open questions on questionnaire, we found that the general opinion of participating students about *ActivMathComp* was largely favourable. Classified as very positive the whole teaching method. Said that it gives more time to practice and more support. Many students stated that it matched their expectations.

In comments many students refer to it as: Positive experience/ excellent initiative; A good way to learn/understand mathematics; Should continue; Different/innovative; Nice atmosphere; Excellent results/ good performance; Well structured; Commitment to learn; Commitment to teach; More motivating; Force students to commitment; Great teacher performance; Allow students to find their difficulties; Help to apply concepts; Would attend again; Hope that ideas of this experience are divulgated to other teachers and other courses.

Research Question 3: Students’ vision about the ILDs

Participating students evaluated the Interactive Learning Documents (ILDs), created specifically for this research, indicating their degree of agreement with the following statements:

- The material was well organized.
- It was easy to find a subject/formula in the PDF's.
- The standardization/uniformity of material was helpful.
- The fact that the material was very uniform disrupted the visual memory. This means: could you memorize the theorem X that appeared on the right corner of that page? (As it happens when you write on paper ...).

Table 4. Number of participants agreeing with statements about the ILDs.

	N.O.	Nothing			Medium			A lot		
	0	1	2	3	4	5	6	7	N	Average
Well organized						5	3	7	15	6.1
Easy to find a subject						3	7	5	15	6.1
Uniformity helpful					2	2	6	5	15	5.9
Bad visual memory			2	2	2	4	3		13	4.3

According to Table 4, all students agree that the materials are well organized and it is easy to find a subject/formula in the documents. About the usefulness of the standardization of material all students agree except 2 that nor agree nor disagree. Most students state that the fact that the material is very uniform is bad for visual memory, although 4 disagree and 2 are indifferent.

Answering the question: "From all that was addressed before, what had the biggest impact in your success/failure?" four students referred the ILDs.

Research Question 4: Students' vision about the weekly online Quizzes

In questionnaire, participating students indicated their degree of agreement with the next statements about the weekly online quizzes:

- The fact that the quizzes provided extra points if we pass seemed positive to me.
- The quiz forced to regularly study the subject.
- The quizzes were important as preparation for the three official tests of MA1.
- The existence of quizzes was positive.

Table 5. Number of participants agreeing with statements about the quizzes.

	N.O.	Nothing			Medium			A lot		
	0	1	2	3	4	5	6	7	N	Average
Give extra points					1	1	3	10	15	6.5
Force to study						1	5	9	15	6.5
Prepar. to assessment						2	4	9	15	6.5
Positive							3	11	14	6.8

As to being positive the fact that the quizzes provide extra points, 14 students agreed and 1 did not agree nor disagree, see Table 5. Everyone agrees that quizzes force students to study regularly the subject and that they were important in preparing for the three official tests of MA1. Students unanimously classified the existence of quizzes as highly positive.

When questioned about why the quizzes were beneficial (or not), all students answered positively. The most mentioned comments were that due to quizzes they "study more often" and "get feedback" about their level of understanding of the subject. Some students also mentioned quizzes make them "be updated on the subject" and "get preparation for the other tests". They also appreciated to "get extra values" for it.

Answering the question: "From all that was addressed before, what had the biggest impact in your success/failure?" five students referred the Quizzes (the biggest score).

In short, the quizzes get a strongly positive classification by the students and in the focus groups students also showed that the quizzes were very important for them.

Threats to validity

Participants' motivation is a concern source, since it may be different from the motivation of the comparison group students, due to their personal characteristics or because of being in a non-ordinary class; we studied it but is impossible to totally measure motivation. Teacher' motivation is another possible problem, since the teacher of the class was the responsible for *ActivMathComp* and for this research is difficult to avoid of having a special enthusiasm into that class. Another concern source is the small number of students in *ActivMathComp* and its difference to the number of comparison group students (16 vs. 519). The small number of participants may affect relationships in classes, may affect statistical results (even though we tried to avoid it during the statistical study) and do not allows to extrapolate the results to higher samples.

Conclusions

ActivMathComp students got higher grades and higher success rate than the others students of MA1. This does not give total confidence to ensure that *ActivMathComp* is more effective than the traditional approach because there are some treats to validity like the small number of participating students, the motivation of students and/or the teacher. However it is a positive argument to its effectiveness.

Students' evaluation of *ActivMathComp* was strongly positive stating that liked the active lessons, the whole teaching method and all the materials. Nearly all answered that would return again (if went again to MA1) and that it matched their expectations. Their evaluation of the Interactive Learning Documents (ILDs) and of the Quizzes was also highly positive.

Thus, is necessary a deeper research using *ActivMathComp* with all the students of the course (now is possible since every student has a personal laptop) and with all the professors to get more reliable research about its effectiveness. The quizzes were identified as a relevant part of *ActivMathComp.* Its implementation, even independently of *ActivMathComp,* should also be studied.

Note

ActivMathComp documentation and materials are available on the website: http://sandragasparmartins.wordpress.com/. The PhD thesis (Gaspar Martins, 2013) that gave rise to this article is available in: http://hdl.handle.net/10362/9675 .
[and include in References:]

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Appendix A. Translation of the text in figures

Figure1
02 Propositional Algebra
Chapter 01: Logic, Set Theory, Induction Method

Conjunction
$p \wedge q$
conjunction of p and q;
p and q

p	q	$p \wedge q$
T	T	…
T	F	…
F	T	…
F	F	…

(True only if p and q are both True)

Example:
(4 is even) ∧ (3+2=1)
... ∧...
...
… I checked my answers in the book.

Properties of conjunction
1. $a \wedge b$... $b \wedge a$
(commutative property)
2. $(a \wedge b) \wedge c$ … $a \wedge (b \wedge c)$
(associative property)
3. $a \wedge T =$…
(T is the identity/ neutral element of ∧)

4. $a \wedge F = \cdots$
 (F is the absorbing element of ∧)
5. $a \wedge a = \cdots$
(idempotence property)
6. $a \wedge \neg a$=…
(non contradiction principle)
... I checked my answers in the book.

Figure 2
Properties of integrals II
 12. f odd ⇒ ...
13. f even ⇒ ...

Figure 3
19. Suppose that you want to make a cylindrical tank to contain 1000 m^3 of water, using the least material possible. Suppose that the material of the top and the bottom is the same that the material of the wall. Which are the tank best dimensions?

Figure 4
06 Special limits
Chapter 03: Sequences
Let's look carefully to the sequence: ...
It's obvious that this sequence is **as close as you want from…**

And now to the sequence:
 It's obvious that this sequence is **as close as you want from…**

Figure 5
04 Modulus
Chapter 02: Real Numbers
4. What are the numbers whose modulus is greater than 3?
a) Choose between this ones:
b) Represent this set in the real line:
c) Represent it as an interval or a union of intervals:
d) Other representation:...

 Can you generalise?
$|x|>a$, $(a>0)$
 is equivalent to:...

Figure 6
01 Definition of Antiderivative
Chapter 09: Antiderivatives
 3. Calculate:
P(f(x)) means the antiderivative of f(x).