On Cost Overruns in Procurement

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Abstract

We consider auctions/tenders for the procurement of goods and services in the setting where the potential contractors face ex-post risks that may lead to cost overruns. The contractors have limited access to credit and are protected by limited liability. We identify the trade-offs that the procurement agency faces in such settings and show that the procuring agency minimizing the expected costs of the project greatly benefits by allocating a share of the award ex-ante, at the time of contracting, with the remainder due ex-post, after the completion of the project.

1 Introduction

Procurement of goods and services has attracted much attention in the economic literature because of the volume of transactions it generates. The procurement by federal, state and local governments is the USA accounts for more than 10% or GDP. The goods and services procured range in complexity and cost, as a result a variety of procurement mechanisms are utilized. The rights to provide complex goods, like new buildings, infrastructure or weaponry are allocated at government tenders. These involve several potential contractors engaged in the consultations about the objectives of the procurer, building prototypes, estimating production costs and finally submitting the offers at the tender where the price is determined by competitive bidding. In USA, Federal Acquisition Regulation strongly encourages the use of such tenders and particularly open competitive bidding whenever possible.[1] Private sector engages in similar tender process when building or renovating homes.

Procurement tender can be analyzed along the same lines as an auction, except that the roles of the sellers and the buyers are reversed. The buyer in, say, real estate auction possesses private information about the value of the house. The contractor that participates in a government tender possesses private information about its own cost of providing the object, say constructing a new bridge.[2] The seller seeks to

[1] https://www.acquisition.gov/far/

[2] The motivation for why the cost of construction can be thought of as private information of the contractor is given in the description of the model.

maximize the sale price of the house, analogously, the agency responsible for the procurement minimizes the cost of construction. The methodology developed for the analysis of auctions can, indeed, be applied directly to the procurement settings if the cost of the project is known with certainty to the potential contractors at the time of the tender. In fact, quite to the contrary, neither the true value of an old house sold at an auction, nor the cost of constructing the bridge, is known at the tender and is discovered by the winner much after the bidding is over.

Here lies a distinction between the auction and the tender. If the winner of the real estate auction discovers later that the value of his house is lower because it is more expensive to renovate, the winner cannot ask the seller for a compensation. If the winner of the procurement tender learns later that the bridge is impossible or unprofitable to complete given the budget, the contractor abandons the project protected by the limited liability.[3] The government is then left to finish the construction at an extra cost. The buyer in the procurement tender therefore suffers from the bidders flawed cost estimates, whereas the seller in the auction is impervious to similar mistakes of the bidders.

The flaws in the costs estimates, that lead to the cost overruns as the project unfolds can be very substantial. Peck and Scherer (1962) estimate that for U.S. defence programs development cost exceed the original predictions by 220 percent on average. Cost overruns are also prevalent in the building industry. "Big Dig" – a highway tunnel under the central part of Boston has been constructed at the cost of 8.6 billion dollars over the original estimate. Opera House in Sydney took extra ten years to complete, went through numerous design revisions with an intent to lower the cost and yet exceeded the original budget by more than 14 times. The responsibility for the cost overrun can lie with the procurer—unexpected change of objectives, the contractor—inadequate design, or "third party"—geological risks, shocks to the material prices, new environmental and building regulations. As is often the case, the responsibility is hard to assign and remains a subject of the heated debate for years, see Boston Globe on the issues surrounding Big Dig project.[4] Bajari and Tadelis (2001) observe that the majority of the building contracts in the USA are either cost plus or fixed price contracts. Banerjee and Duflo (2000) observe that same in the Indian software industry oriented for the USA clients. Banerjee nd Duflo "contracts that target the sourse of the cost overrun are probably unenforsable."

We provide an original model of procurement process with asymmetric information about the cost of the project. The cost of the project is subject to the shock which is realized after the construction has started. The value of the cost overrun is private information of the winner of the tender. This precludes the use of cost plus contracts, where the burden of the cost overrun is transferred to the buyer, see Bajari and Tadelis

[3] In the US during 1990-1997 more than 80,000 contractors filed for bankruptcy leaving behind unfinished private and public construction projects with liabilities exceeding $21 billion. (Dun & Bradstreet Business Failure Record).

[4] http://www.boston.com/news/specials/bechtel/part_1

(2001). Our analysis thus concentrates on so called fixed-price contracts, with the level of the price determined by competitive bidding. This form of the contract is most commonly used in the government defence procurement, see Fox (1974), McAfee and McMillan (1986) and in the public infrastructure procurement, see Tadelis and Bajari (2006). The government allocates the project via the second lowest bid tender.[5] The overall amount of the award that the winner is set to receive is determined at the time of the tender. The novelty of the approach is that the award can be allocation in proportions, one part ex-ante—before the construction starts and the remainder ex-post—after the completion of the project. The proportion is known to the bidders at the time of the tender and certainly affects their bidding behavior. We consider two settings, in one the firms have unlimited budgets to cover any possible costs, in the more general one the firms have limited access to credit. The budget constraint constitutes another aspect of the contractor's private information in the spirit of Che and Gale (1998). We assume that the contractors are protected by limited liability therefore our work also contributes to the new and growing field of mechanism design without commitment.

As we show, the following fundamental tensions arise. The higher is the proportion of the tender paid ex-ante, the lower are the bids. The bidders protected by the limited liability are not afraid to walk away from the project in case of the large cost overrun and bid quite aggressively. In the extreme case of full ex-ante payment, the bidders bid their private costs and abandon the project whenever the shock causes cost overruns. The buyer contracts to complete the project at the lowest possible price but faces the additional costs due to incomplete projects more often. We assume that it is more costly to complete the project for the buyer than for the winning contractor. With fully ex-post payment the contractors are more exposed to the risks associated with the shocks and inflate their bids accordingly. They, however, have the strongest incentives to complete the project. In case of unlimited budgets we show that it is optimal to pay the whole amount of the award ex-post. When the budgets are limited fully ex-post payment often leads to incomplete projects because the winner has no funds to finish the project. At the same time if the award is fully paid ex-ante the winner has no incentive to finish the project in case of the cost overrun. We show that in the case of limited budgets the optimal proportion is interior, it is optimal to pay a fraction $r \in \left(0, \frac{1}{2}\right)$ of the tender ex-ante and the remainder $(1-r)$ ex-post if and when the project is completed.

The rest of the paper is organized as follows. Section 2 summarizes the relevant literature, Section 3 presents the model, Section 4 deals with the special case of unlimited budget, Section 5 with the case of "lean budget" where each bidder just has enough own funds for the initial investment. The special cases are not only of independent interest but also serve as building blocks of the equilibrium construction

[5] While the second lowest bid format is rarely used for procurement, we use it as a notationally simpler equivalent to an open descending price format that is commonly used. The results qualitatively are likely to be the same.

for the general two dimensional case of arbitrary budgets presented in Section 6. Section 6 also contains our main result. Section 7 extends the results for unlimited and lean budgets to the case where shocks z arrive from distribution with continuous c.d.f. F_z. The equilibrium strategies then cannot be obtained in closed form but optimal sharing schemes are the same as for binary distribution of shocks in sections 4 and 5. We expect to generalize the main result in Section 6 to the general continuous distribution of the shocks.

2 Related Literature

The asymmetric information literature on procurement is quite large, see the books by Laffont and Tirole (1993) and McAfee and McMillan (1988) for surveys. The modern approach to the procurement problem recognizes the elements of adverse selection, the contractor has better information about the production cost than the buyer, and of moral hazard, the contractor may reduce the cost of production by exerting more effort. The approach to the adverse selection is the usual screening, the buyer offers a menu of contracts from which the seller selects a particular one thereby reducing the informational asymmetry. To our knowledge, the screening stage is always modelled as one on one interaction, that is the buyer has already selected the contractor for the project and now the problem is reduced to providing incentives for this particular contractor.

Another approach to informational asymmetry is competitive bidding. Here of course, there are multiple potential contractors who posses private information about the production cost. The competitive bidding in the setting where the contractors know with certainty what the cost will be before they start production is equivalent to an auction and can be analyzed along the lines of Myerson (1981) or Riley and Samuelson (1981) optimal auction, (see also Manelli and Vincent (1995) for optimal procurement mechanism when goods vary in quality.)[6] This setting is however, unrealistic. Ashley and Workman (1986) in a survey of contractors and buyers in USA building industry report that project engineering must be 40-60% complete to establish the reasonable estimate for the cost.

McAfee and McMillan (1986) consider a model similar to ours where the costs are subject to ex-post shocks but also may be reduced by the winner's effort. The buyer therefore faces both adverse selection and moral hazard. McAfee and McMillan consider bidding for a linear incentive contract that factors in both the cost (assumed observable ex-post) and the winning bid and derive the optimal contract in this class. The contract in McAfee and McMillan therefore combines the elements of the fixed price and cost plus contracts. Such contracts are infeasible in our setting as both the original estimate and the final value of the cost remain the private information of

[6] The literature on auctions with budget constrained bidders is small, see Che and Gale (1998) and (2000), Che, Gale and Kim (2011), Pai and Vohra (2009).

the winner. Parlane (2003) also deals with bidding in the model with potential cost overruns and bidders protected by limited liability. She shows that among all efficient mechanisms in which only the winner gets paid, the lowest bid tender leads to the highest expected price, thus minimizing the chances of bankruptcy. In our setting where the bidders are budget constrained neither the first nor the second lowest bid tender is efficient.

Piccione and Tan (1996) consider a model in which ex-ante symmetric potential contractors invest in the cost reduction technology and then compete for the procurement contract. The cost can be further reduced by exerting effort with no exogenous shocks. If the buyer is able to commit to the procurement mechanism before the investment stage the first best solution can be implemented by either first or second price auction under quite general conditions. If the buyer chooses the mechanism after the initial investment stage the level of investment is suboptimal. Arozamena and Cantillon (2004) consider a similar model, however, the firms are ex-ante heterogeneous, investment is only made by one firm and the level of investment is made observable introducing asymmetry in the competition. If investment affects which firm is the most efficient then the first-price auction will induce less investment than second-price auction. In all of these papers the contractors have sufficient budgets to cover all possible costs and it is *assumed* that the award is paid ex-post.

Burguet et. al. (2006) study procurement in the setting where the bidders are symmetric with respect to both ex-ante and ex-post costs, but have privately known budgets. With limited liability and budget being the only screening variable incentive compatibility implies that win probabilities are monotonically decreasing with the budgets, thus an auction allocates the contract to the least financially solvent contractor. In our setting, naturally, privately known budgets affect the bidding strategies but those mainly depend on the privately known costs, see the equilibrium construction in Section 6.

Contract theory literature that deals with one on one relationship between the buyer and the contractor is huge. Here we concentrate on the papers that deal with procurement and issues of cost overruns. Again with few exceptions all of this literature assumes no credit constraints. Bajari and Tadelis (2001) build the model in which there is no role for the ex-ante asymmetric information, however, the design may be subject to ex-post changes. The contractor possesses private information about the cost of the change of the original design. The buyer ex-ante decides on the completeness of the design. With more complete design the likelihood that it will need to be renegotiated ex-post is smaller, which reduces the ex-post cost of the buyer. Providing more complete designs is, however, costly ex-ante. Bajari and Tadelis compare cost plus and fixed price contracts and show that simple projects will be procured via fixed price contracts and will have a high level of design completeness. More complex projects will be procured at cost plus contracts and will have low level of completeness. Crocker and Reynolds (1993) provide an empirical study of the effects of various types of contracts used in defence procurement.

In Riordan and Sappington (1988) the buyer either uses contingent prices or can commit to negotiate a price in the second period once uncertainty about the shock is realized. It is often difficult to provide a complete contract that will cover any contingency and impossible to do so if the costs are not verifiable like in our model. Committing to negotiate a price in the future may be suboptimal since the contracts will be submitting very low bids trying to lock in the contract before the shock is realized and take advantage of the situation to increase the price at the renegotiation stage.

Tirole (1986) in an incomplete contract setting examines the ex-ante investment in cost reduction by the contractor. Parties bound by an incomplete contract have an incentive to renegotiate after acquiring the information about the shock. If the contractor's investment is not observed by the buyer, in a vast majority of bargaining schemes, the firm invests less than it would under symmetric information and a complete contract. If the investment can be observed by the two parties that jointly decide on the firm's investment level (and on the investment sharing rule). the mutually agreed-on investment may be lower than its level under unobservability, higher than the first-best level, or intermediate between the two values.

3 Model

The government, later a buyer, needs to realize a new project that once completed it values at V. There is a number N of contractors/firms that have a capacity to complete the project. Each contractor i is given the right to examine the requirements of the project to be completed and forms its estimate of the cost of the initial investment, c_i. This cost is private information of firm i, it is unverifiable and cannot be contracted upon.[7] The buyer, though, can ascertain if the necessary initial investment has been made.[8]

Any contractor that is awarded the contract and invests in it faced ex post risk. Before completion, but after the initial investment of c_i, the winner may have to incur extra costs $Z > 0$ with probability of such an adverse event occurring $p > 0$. [9] These costs can arise due to a management oversight, an adverse shock to input

[7] We implicitly assume here that each contractor is endowed with exogenously given amount of capital and labor and that contractors are symmetric in that respect. The asset of the contractor that is responsible for the private information in the model is the managerial talent of the contractor. This talent is intangible and unverifiable, cannot be therefore used as collateral, but can reduce the cost of the project. A large construction project involves coordination of work of hundreds of independent sub-contractors and managerial costs can be very substantial. Bajari and Tadelis (2001) provide an excellent description of construction industry in the USA and the work on Getty Art Centre in Los Angeles.

[8] This assumption is needed for off equilibrium considerations. In equilibrium the contractor is provided with incentives to make an initial investment.

[9] Parlane (2003) considers similar model but there the shock occurs without any investment, purely as the time unfolds. It is left unspecified why the tender is not delayed untill all the uncertainty

costs, or some other unforeseen contingencies. We assume that these costs are also not verifiable and so cannot be contracted upon and that they are the same both in magnitude and probability for any firm. The latter assumption is made solely for simplicity of exposition. The firm may decide not to cover these extra costs, in which case the project will not be completed.[10]

The buyer uses a second-lowest sealed bid auction to allocate the contract, that is, it asks for each contractor to make a bid, awards the contract to the one with the lowest bid, and sets the value of the award β equal to the second lowest bid. The crucial choice of the buyer is to decide on the share of the award to be transferred at the initial investment stage, $r \in [0, 1]$. Thus, once the tender finishes and the award is set at β, the buyer pays to the winner $r\beta$, and once the project is completed, it pays the remainder $(1 - r)\beta$. We assume that if the shock hits and the winner decides not to cover extra costs, the buyer is not obligated to pay the ex-post part of the award. If she decides to complete the project it has to incur extra costs of $X > Z$.[11] The inequality reflects possible extra costs (in addition to Z) from time delays, inadequate expertise, and other factors.

Each contractor faces a "hard budget" constraint when making each investment. Contractor i initially has m_i in cash, which is her private information. For each investment it wants to make, i has to have enough cash to cover it. We assume that each contractor has limited liability in the sense that it cannot be forced to make an unprofitable initial or additional investment. Thus, each contractor has a two-dimensional private information (c_i, m_i), independently drawn from other firms from the same cumulative distribution F. Thus, the common knowledge includes values of V, X, Z, p, and c.d.f. $F[c, m]$ on $\left[C^{\min}, C^{\max}\right] \times \left[S^{\min}, S^{\max}\right]$.[12] We assume that $V > C^{\max} + pZ$ and $X < V$, so that it is always optimal for the buyer to procure for the project and complete an incomplete project if it comes to that.

Finally, the objective of the buyer is to maximize its value from having the project completed considering the payments to be made. Given our assumptions it amounts to minimizing the expected cost of the project, inclusive or all the payments the buyer faces. The objective of each contractor is to maximize its profits. Both the buyer and contractor are risk-neutral and have the same value for money.

A trivial immediate observation is that it is optimal for the buyer to make the

is realized. Our model better captures the feature of the reality that for the contractor to realize the actual cost of the project she has to actually work on it.

[10] This is an extreme form of limited liability, a more realistic assumption that the buyer can enforse a contract by incuring a cost $\sigma > 0$ leads to the same qualitative results.

[11] Implicitly we assume that there is another tender process that results in some contractor being chosen to complete the project. It is important that the original contract is not renegotiated. In addition, the winner of the original tender is excluded from participation in the second tender.

[12] At this stage we make no further assumptions on $F[c, m]$. Deriving the expected cost of the project we will assume that $m_i \geq c_i$ for each contractor. We will discuss this assumption in more detail after the equilibrium is derived. Roughly $m_i \geq c_i$ in our model is guaranteed if the contractors have access to competitive banking sector.

incentives for the initial and additional investments the strongest, that is, pay $r\beta$ if and only if the necessary initial investment occurs, pay $(1-r)\beta$ if and only if the additional investment is made and the project is actually completed, and pay nothing otherwise. We will show that in equilibrium the winner indeed has the incentives to invest c_i.

Thus, the timing of the game can be represented as follows. At time 0, the buyer announces r, each contractor i learns (c_i, m_i). At time 1, the tender is being conducted, the contractors submit bids, the government selects the winner, say bidder j, the size of the award β is determined. Shortly thereafter, the first installment of $r\beta$ is made and the initial investment of c_j is made. At time 2, the shock—the need of an extra investment—may hit. If it does not, the project is completed and the ex post payment is made. If the shock hits, the winner decides whether to make the extra investment. If she does not, she does not receive an ex post payment, and the buyer pays X to the third party, otherwise the project is completed and the ex-post payment is made. The timing is therefore as follows.

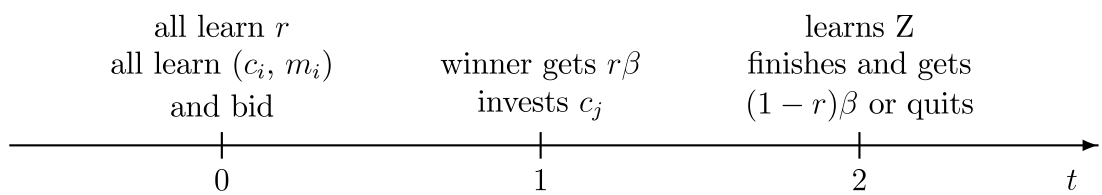

Figure 1: Timeline

Our model therefore extends the conventional screening model of procurement in three dimensions. We add post-allocation shocks, limited liability and budget constraints. The buyer's sole control variable is r, the idea is that such modest departure from the standard ex-post payment scheme may be well received by the industry regulators.

We further provide some illuminating examples where the distributions $F(c, m)$ is of special interest. The equilibria derived in these examples are later used as building blocks for the equilibrium construction in the general case.

4 Unlimited Budgets

In this section we examine the simplified problem where for each firm the budget constraint is never binding $s_i > c_i + Z$. To gain the intuition we start with considering the regimes of fully ex-post and fully ex-ante payments. Then we consider the payment scheme that corresponds to arbitrary r. We show that with unlimited budgets fully ex-post payment is optimal. Note that this case corresponds to degenerate distribution with $s_i = S > C + Z$ for every bidder.

4.1 Full Ex-post Payment

We first examine the case of fully ex-post payment and derive the equilibrium bidding strategies. Consider bidder i with cost c_i. Suppose this bidder submits bid b. The amount of the awarded tender, $\beta \geq b$, by the rules of the tender. To calculate the expected profit and the optimal value of b consider the following realizations. If total cost $Z + c_i \leq \beta$, it is optimal to complete the project which leads to the profit of bidder $i : \pi_i = \beta - Z - c_i$. If $Z + c_i > \beta$, then $\pi_i = \max\{-c_i, \beta - Z - c_i\}$. Since the amount c_i is sunk before bidder i learns Z, when $\beta < Z$ the winning bidder optimally abandons the project and the profit $\pi_i = -c_i$. If $\beta \geq Z$, the project will be completed and the profit is again $\pi_i = \beta - Z - c_i \geq -c_i$. Thus if $b > Z$ the profit of bidder i conditional on winning the tender is either $\beta - c_i$ or $\beta - c_i - Z$ depending on whether the shock hits or not. To derive the expected payoff of bidder i, consider all the bidders but bidder i, find among them the bidder with the lowest cost and denote with $\beta_1(c)$ his bidding function. Assume that β_1 is strictly increasing and continuous such that inverse $\beta_1^{-1}(b)$ is well defined. Denote with $G(\beta_1)$ the c.d.f. of β_1. Introduce

$$\pi(c, b) = \int_b^{\beta_1^{-1}(\bar{c})} (\beta_1 - c)\, dG(\beta_1).$$

If bidder i with cost c_i submits bid $b > Z$, her expected payoff

$$E[\pi_i] = (1-p)\,\pi_i(c_i, b) + p \int_b^{\beta_1^{-1}(\bar{c})} (\beta_1 - Z - c_i)\, dG(\beta_1) =$$

$$E[\pi_i] = \int_b^{\beta_1^{-1}(\bar{c})} (\beta_1 - pZ - c_i)\, dG(\beta_1)$$

Therefore it is optimal to bid $b^* = c_i + pZ$ as long as $b^* \geq Z$, Note that at $b^* = Z$, $c_i = (1-p)Z$ that is the optimal bid

$$b^* = c_i + pZ \text{ for } c_i \geq (1-p)Z$$

Now, if bidder i with cost c_i submits bid $b < Z$ the awarded tender β can be above or below Z. If $\beta \geq Z$ as before there are two realizations of the profit $\pi_i = \beta - c_i$ or $\pi_i = \beta - c_i - Z$ depending on whether the shock hits or not. If $\beta < Z$ and there happens to be a shock Z, extra investment will not be undertaken and bidder i will optimally leave the project with the profit $\pi_i = -c_i$. The expected payoff of bidder i who submits bid $b_i < Z$:

$$E\pi_i = (1-p)\,\pi_i(c_i, b) + p\left[\int_b^Z (-c_i)\, dG(\beta_1) + \int_Z^{\beta_1^{-1}(\bar{c})} (\beta_1 - Z - c_i)\, dG(\beta_1)\right]$$

collecting the terms and simplifying:

$$E\pi_i = (1-p)\int_b^{\beta_1^{-1}(\bar{c})} \left(\beta_1 - c_i/\left(1-p\right)\right) dG\left(\beta_1\right) + p\int_Z^{\beta_1^{-1}(\bar{c})} \left(\beta_1 - Z\right) dG\left(\beta_1\right)$$

The optimal bid is therefore

$$b^* = c_i/\left(1-p\right) \text{ for } b^* < Z.$$

Note again that at $b^* = Z$, $c_i = (1-p)\,Z$ so that for bidders with cost $c < (1-p)\,Z$ it is optimal to bid according to $b^* = c_i/\left(1-p\right)$. The optimal bid overall

$$b^* = \begin{cases} c_i/\left(1-p\right) & \text{if } c_i < (1-p)\,Z \\ c_i + pZ & \text{if } c_i \geq (1-p)\,Z \end{cases}$$

Note also that at $c_i = (1-p)\,Z$.

$$c_i + pZ = c_i/\left(1-p\right)$$

so that the derived optimal bidding function is increasing and continuous but involves a kink at $c = (1-p)\,Z$.

Now let's examine the expected cost of the project. Denote with c_1 the cost of the winner, the lowest cost. When $c_1 \geq (1-p)\,Z$ in equilibrium the project is always completed. In case the shock hits, the contractor compares the extra cost of completing the project Z with the payment that will result only if the project is completed. This payment exceeds $c_1 + pZ$ and the contractor optimally completes. The cost of the project in such realization is $c_2 + pZ$, where c_2 is the second lowest cost. The expected cost over these realizations is $E\left[c_2\right] + pZ$. In case $c_1 < (1-p)\,Z$ and $\beta > Z$ the project is also completed by the winner. Note that since the bid of the winner $c_1/\left(1-p\right) < c_1 + pZ$ the expected cost of the project is lower than $E\left[c_2\right] + pZ$. In case $\beta < Z$ the project is not completed, but c_1 is invested from bidder 1 funds. The buyer has to pay X to complete the project.

Overall the project costs at most $E\left[c_2\right] + pZ$ when completed by the winner and X when completed by the buyer when the tender is paid ex-post.

4.2 Full Ex-ante Payment

Now suppose the payment is made ex-ante. The winning bidder invests c_i to carry on with the project until the shock is realized. When the realized shock is 0 the project is competed and the winner of the tender gets the payoff $\beta - c_i$. When the realized shock is Z, the project is not completed and the winner of the tender quits the project. Since the initial costs and the value of the shock are not verifiable the winner optimally expropriates the residual $\beta - c_i$ no matter what the shock is. Thus i's expected payoff conditional on winning is $\beta - c_i$. The optimal bid

$$b^* = c_i \text{ for all } c_i.$$

The buyer obtains the project at the second highest cost but whenever the shock hits they have to complete the project for the cost $X > Z$.

$$E[\beta]^- = E[c_2] + pX,$$

In the realizations where with ex-post payment the project cost is X it is $E[c_2] + X$ with ex-ante payment. In all other realizations the expected cost with ex-post payment

$$E[\beta]^+ \leq E(c_2) + pZ \leq E(c_2) + pX = E[\beta]^-$$

Thus with unlimited budgets ex-post financing dominates ex-ante if $X > Z$.

4.3 Fixed Share Scheme

More generally suppose the fraction r of the award β is being paid ex-ante and the remainder upon completion of the project. Even though the bidders do not face hard budget constraints they may be unwilling to finish the project if the ex-post portion of the award $(1-r)\beta < Z$. For bidder i with cost c_i in case the shock hits the payoff is $\pi_i = \beta - c_i - Z$ if $\beta(1-r) \geq Z$. If $\beta(1-r) < Z$ bidder i will optimally quit the project with the payoff $\pi_i = r\beta - c_i$. Suppose bidder i with cost c_i bids $b \geq Z/(1-r)$ and $\beta_1^{-1}(\cdot)$ is the inverse of the bidding function of the opponent with the lowest cost. Bidder i's expected payoff is:

$$\int_b^{\beta_1^{-1}(\bar{c})} (\beta_1 - c_i - pZ)\, dG(\beta_1)$$

which is again optimized with $b^* = c_i + pZ$.

When bidder i with cost c_i bids $b < Z/(1-r)$ her expected payoff is

$$E\pi_i = (1-p)\,\pi_i(c_i, b) + p \int_b^{Z/(1-r)} (r\beta_1 - c_i)\, dG(\beta_1) + p \int_{Z/(1-r)}^{\beta_1^{-1}(\bar{c})} (\beta_1 - c_i - Z)\, dG(\beta_1).$$

Collecting the terms that are affected by b

$$E\pi_i = \int_b^{Z/(1-r)} ((1 - p(1-r))\,\beta_1 - c_i)\, dG(\beta_1) + \int_{Z/(1-r)}^{\beta_1^{-1}(\bar{c})} (\beta_1 - c_i - pZ)\, dG(\beta_1)$$

For $b < Z/(1-r)$ with $q = 1 - r$ the optimal bid is then $b^* = c_i/(1-pq)$. Introduce

$$c^* = \frac{Z}{q} - pZ.$$

The optimal bid overall

$$b^* = \begin{cases} c_i/(1-pq) & \text{if } c_i < c^* \\ c_i + pZ & \text{if } c_i \geq c^* \end{cases}.$$

Note that at c^*, $c^* + pZ = c^*/(1-pq)$, and $b^*(c^*) = Z/q$ so that the bidding strategy involves a kink at c^*, see Figure 2.

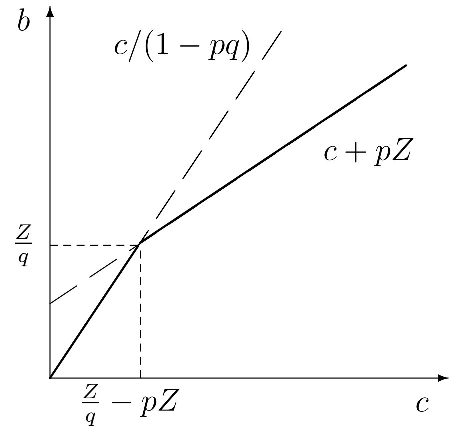

Figure 2: Bidding strategy when the bidder faces "willing" to finish constraint

4.4 Expected Cost of the Project

Note that the buyer always finishes the project even if the contractor abandons the projects after the cost overrun. The government naturally only pays the ex-ante portion of the award in such event. Note that the second lowest cost c_2 determines the amount of the award. Recall that X is the extra cost of completion if the winner abandons the project after the cost overrun. Suppose $c_2 < c^*$. Then in case of no cost overrun, with probability $1 - p$, the project is completed and costs $c_2/(1-pq)$ to the government. In case of cost overrun, with probability p, the project is optimally abandoned by the contractor, since $b(c_2) < Z/q$. The contractor then receives only the ex-ante part of the award $rc_2/(1-pq)$. In addition, in these realizations the buyer invests $X > Z$ to finish the project. Thus conditional on $c_2 < c^*(r)$ the expected cost of the project is $c_2 + pX$. If $c_2 \geq c^*(r)$ the project is completed regardless of the cost overrun, since $b^*(c_2) > Z/q$, and costs $c_2 + pZ$ to the buyer. Denote with $H(c)$ the distribution of the second lowest cost, the second lowest order statistics from n draws. The expected cost of the project:

$$E[C] = H(c^*)\left[E[c_2|c_2 < c^*] + pX\right] + (1 - H(c^*))\left[E[c_2|c_2 \geq c^*] + pZ\right]$$

Since $H(c^*)E[c_2|c_2 < c^*] + (1 - H(c^*))E[c_2|c_2 \geq c^*] = E[c_2]$ the expected cost

$$E[C] = p(X - Z)H(c^*(r)) + E[c_2] + pZ$$

Since H is increasing and $X > Z$ the expected cost is minimized when c^* is minimized, by setting $r = 0$.

Proposition 1 *With unlimited budgets it is optimal to pay the entire award ex-post.*

Corollary 1 *With $X = Z$ any r is optimal.*

Note that the case of $X = Z$ effective assumes that any contractor—the original or the replacement one can fix the cost overrun for the same price. Therefore it may seem that the optimal contract can be written as fixed price with a contingency that in case of the cost overrun the contract switches into cost plus. Indeed in equilibrium the fact that the contractor walks away from the project reveals to all the parties that the shock is Z and with the cost being common knowledge cost plus contract is optimal. This intuition is false, since introducing such a contingency in the contract destroys the incentives of the original contractor to invest c_i. It is important for the result that the original contract is not renegotiated and remains fixed price. In case of the cost overrun it is better to let the contractor walk away and replace him with a new one. This may not always be feasible in defence procurement or major infrastructure projects, but is quite feasible in case of house renovation or car repair.

After the original contractor walks away from the house renovation there is normally more quotes solicited and the tender process (for the works left to do) is repeated. This tender process generates some price. We are not modelling this tender process, instead assume that it generates price X to complete the project. Due to adaptation costs alone $X > Z$, which is what is needed for the result. Section 7 deals with the case when shock z is drawn from an arbitrary distribution F_z. There the fact that the original contractor quits the project does not reveal to the buyer the true value of z. Any other contractor invited to the project knows z but also knows that the buyer does not know it perfectly. We abstain from modelling the interaction between the new contractor and the buyer and assume instead that in case the original contractor left the project and the realized shock is z it costs $z + \delta$ to complete the project. Out of these only the value of $\delta > 0$ is common knowledge and reflects adaptation costs, the effects of delays, etc.

Note that in the auction setting with iid private values the second price auction with correctly chosen reserve price is optimal, Myerson (1981). The analog of reserve price in our setting is a "award floor". The award is constrained from the bottom so that the winner receives $\max\{R, \beta\}$. This amount is also being paid in proportion r ex-ante and $1 - r$ upon completion. Clearly the choice of R will affect the expected cost of the project. It is suboptimal to choose R such that the winner who is willing to finish the project and receive $(1 - r)\beta$ is getting extra compensation. That is optimal $R \leq Z/q$. It is also suboptimal to pay the award that does not provide enough incentives to finish the project, that is $R \geq Z/q$. With $R = Z/q$ if $c_2 \geq c^*$ the project is completed by the winner and the award is $c_2 + pZ$. If $c_2 < c^*$ the project is also completed and the award is Z/q. Since $c^*(r) = Z/q - pZ$, the expected cost of the project can be written as

$$E[C] = pZ + \int_0^{c^*} c^* dH(c_2) + \int_{c^*}^{C} c_2 dH(c_2).$$

Clearly the loss is increasing in c^*, therefore it is optimal to again choose $r = 0$ and $R = Z$. The optimal expected cost with the award floor is $E[C]^* = pZ + c^* H(c^*) + \int_{c^*}^{C} c_2 dH(c_2)$. Without the award floor with the same optimal choice of c^* the expected cost is

$$E[C]^{**} = pZ + p(X - Z) H(c^*) + \int_0^{c^*} c_2 dH(c_2) + \int_{c^*}^{C} c_2 dH(c_2),$$

so that it is optimal to introduce the award floor if

$$p(X - Z) H(c^*) + \int_0^{c^*} c_2 dH(c_2) > c^* H(c^*)$$

Recall that optimal $c^* = (1 - p) Z$. It is then optimal to pay the entire award ex-post and introduce the award floor $R = Z$ if

$$E[c_2 | c_2 \leq (1 - p) Z] > Z - pX.$$

Note that in the second price action optimally set reserve price always improves expected revenue. In particular, a small reserve price always helps. This is not the case here. In our setting a small award floor will almost for sure lead to higher payment to the winner that does not, however, provide him any additional incentives to finish the project. Small award floors are therefore worse than none. Because of such inflexibility with the choice of the award floor they only improve expected revenue if the loss from incomplete projects are high enough,

5 Budget Constrained Bidders

In this section we assume that the bidders' types are two dimensional. Bidder i's type is $[c_i, m_i]$, where m_i is i's financial reserves. This m_i constitutes a "hard" budget constraint. Bidder i in case she wins the tender can only use the portion of the tender allocated ex-ante and her own reserves m_i to cover the cost of the project. The initial investment of c_i is feasible if

$$m_i + r\beta \geq c_i$$

After initial investment in case shock Z is realized the winner will continue with the project if two conditions are met. First, if $Z < (1-r)\beta$ the ex-post part of the award covers the shock and the winner will prefer to finish the project. Second, after the initial investment the winner has to have enough funds to finish the project: $m_i + r\beta - c_i \geq Z$

$$Z \leq \min\{(1-r)\beta, r\beta + m_i - c_i\} \tag{1}$$

We will derive the equilibrium bidding strategies in this setting. Even though we restrict our attention to the second price auction, the resulting equilibrium is quite involved. We therefore start with a simplified case where $m_i = c_i$ for every bidder. The elements of the equilibrium construction will be later used as building blocks for the equilibrium in the general case.

5.1 Lean Budgets

Suppose $m_i = c_i$ for every bidder. Then condition (1) simplifies to

$$Z \leq \min\{(1-r), r\}\beta$$

and there are two cases to consider. With $r < \frac{1}{2}$ only the "able" to finish constraint $Z \leq r\beta$ may be binding. With $r \geq \frac{1}{2}$ only the "willing" to finish $Z \leq (1-r)\beta$ constraint may be binding.

Suppose first that $r < \frac{1}{2}$. In case of a negative shock if $\beta \geq Z/r$ then the bidder will be able to cover the shock, finish the project and receive $\beta - Z - c_i$ as payoff. If $\beta < Z/r$ the bidder will initially invest, will not be able to cover the shock, will lose original c_i, and further quit the project with payoff $r\beta - c_i$. If $b < Z/r$ both options are possible. If $b > Z/r$ only $\beta \geq Z/r$ is possible. Suppose bidder i with cost c_i submits bid $b_i \geq Z/r$. The project is always finished and the expected payoff of bidder i

$$E\pi_i = \int_{b}^{\beta_1^{-1}(\bar{c})} (\beta_1 - c_i - pZ)\, dG(\beta_1),$$

The optimal bid is then $b^*(c_i) = c_i + pZ$ if $b^* > Z/r$. Again comparing such b^* with Z/r introduce

$$c''(r) = \frac{Z}{r} - pZ.$$

For bidders with $c_i \geq c''(r)$ it is optimal to bid according to $b^*(c_i) = c_i + pZ$.

Next suppose bidder i with cost c_i submits bid $b_i < Z/r$. The expected payoff

$$E\pi_i = (1-p)\,\pi_i(c_i, b) + p\int_{b}^{Z/r} (r\beta_1 - c_i)\, dG(\beta_1) + p\int_{Z/r}^{\beta_1^{-1}(\bar{c})} (\beta_1 - c_i - Z)\, dG(\beta_1)$$

Collecting the terms that are affected by the choice of b

$$E\pi_i = \int_b^{Z/r} \left(\left(1 - p\left(1 - r\right)\right)\beta_1 - c_i\right) dG\left(\beta_1\right) + \int_{Z/r}^{\beta_1^{-1}(\bar{c})} \left(\beta_1 - c_i - pZ\right) dG\left(\beta_1\right)$$

The second integral in the expected payoff is positive and the first integral is maximized by setting $b^*\left(c_i\right) = c_i / \left(1 - p + pr\right)$ if $b^* < Z/r$. With $q = 1 - r$, for $b^* < Z/r$, $b^*\left(c_i\right) = c_i / \left(1 - pq\right)$. Comparing such b^* with Z/r introduce

$$c'\left(r\right) = \frac{Z}{r}\left(1 - pq\right) = \frac{Z}{r}\left(1 - p\right) + pZ.$$

For bidders with $c_i \leq c'\left(r\right)$ it is optimal to bid according to $b^* = c_i / \left(1 - pq\right)$.

Note that for any $r < \frac{1}{2}$, $c'\left(r\right) < c''\left(r\right)$ and for $r = \frac{1}{2}$, $c'\left(r\right) = c''\left(r\right)$. Note also that both $c'\left(r\right)$ and $c''\left(r\right)$ are strictly decreasing in r. With $r < \frac{1}{2}$ the optimal bid is therefore

$$b^*\left(c_i, d_i\right) = \begin{cases} c_i / \left(1 - pq\right) & \text{if } c_i < c'\left(r\right) \\ Z/r & \text{if } c_i \in \left(c'\left(r\right), c''\left(r\right)\right) \\ c_i + pZ & \text{if } c_i > c''\left(r\right). \end{cases}$$

The bidding function above is increasing and continuous and incorporates a flat interval at the level $b^* = Z/r$ for $c_i \in \left(c'\left(r\right), c''\left(r\right)\right)$. It may seem that bidder i who is supposed to submit a bid Z/r is facing a mass point in the bidding strategies of the opponents and will benefit from submitting a slightly lower bid. It is true that doing so will increase i's chances of winning but at the same time increases her expose to the risks of incompletion. After the deviation she may win and only receive $\beta < Z/r$ in ex-ante portion of the award and will lose her initial investment of c_i. It is helpful to remember for future reference that when bidding strategy passes through the level of "able" to finish constraint it involves a flat interval and a kink, see Figure 3.

Suppose now $r \geq \frac{1}{2}$, so that the constraint $(1 - r)\beta < Z$ may be binding. In case of a negative shock if $\beta \geq Z/\left(1 - r\right)$ then the bidder will finish the project and receive $\beta - Z - c_i$ as payoff. If $\beta < Z/\left(1 - r\right)$ the bidder will lose his original investment of c_i and further quit the project with payoff $r\beta - c_i$. If $b < Z/\left(1 - r\right)$ both options are possible. If $b > Z/\left(1 - r\right)$ only $\beta \geq Z/\left(1 - r\right)$ is possible. As in the preceding analysis if bidder i with cost c_i submits bid $b_i < Z/\left(1 - r\right)$ the optimal bid is $b^*\left(c_i\right) = c_i / \left(1 - p + pr\right) = c_i / \left(1 - pq\right)$. If bidder i with cost c_i submits bid $b_i > Z/\left(1 - r\right)$ the optimal bid is $b^*\left(c_i\right) = c_i + pZ$. Note that at $c^*\left(r\right) = Z/q - pZ$

$$c^*\left(r\right) / \left(1 - pq\right) = c^*\left(r\right) + pZ = Z/q$$

so that the derived bidding strategies are strictly increasing and continuous and involves a kink at $c^*\left(r\right)$.

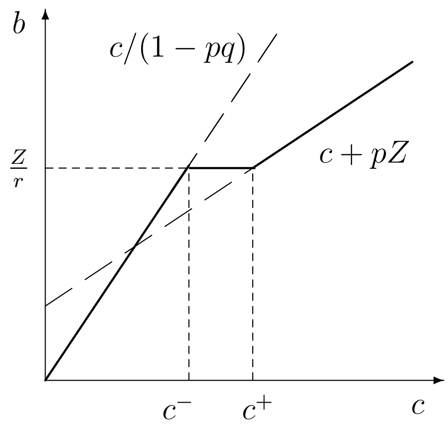

Figure 3: Bidding strategy when the bidder faces "able" to finish constraint

5.2 Expected Cost of the Project

Proposition 2 *With lean budgets the expected cost of the project is minimized by setting* $r = \frac{1}{2}$.

Proof. Since with $r \geq \frac{1}{2}$ the equilibrium bidding strategy is exactly the same as with unlimited budgets, with $r \geq \frac{1}{2}$ to minimize the expected cost it is optimal to set $r = 1/2$. Note that with $r \geq \frac{1}{2}$ the expected cost is strictly increasing in r.

With $r < 1/2$ the project is completed when $b^*(c_2) \geq Z/r$ despite of the cost overruns and is only completed with probability $1-p$ when $b^*(c_2) < Z/r$. In our earlier notations when $c_2 \leq c'(r)$ the project is completed by the contractor with probability $1-p$ and abandoned with probability p. In the latter case the contractor only collects the ex-ante portion of the award $rb^*(c_2)$ and the buyer finishes the project at extra cost X. Therefore conditional on $c_2 \leq c'(r)$ the expected cost of the project is $c_2 + pX$. The contractor finishes the project if $c_2 \geq c''(r)$ and the buyer pays $c_2 + pZ$, and when $c_2 \in (c'(r), c''(r))$ in which case the buyer pays Z/r.

If the second lowest cost, c_2 is distributed according to c.d.f. $H(\cdot)$, the expected cost of the project

$$\begin{aligned} E[C(r)] &= H(c'(r))\left[E[c_2|c_2 \leq c'(r)] + pX\right] + \\ &\quad + \left(H(c''(r)) - H(c'(r))\right) Z/r + \left(1 - H(c''(r))\right)\left[E[c_2|c_2 \geq c''(r)] + pZ\right]. \end{aligned}$$

It is more convenient to minimize the expected cost after $c_2 + pZ$ is added and subtracted in every realization. Since

$$\left(H(c'') - H(c')\right) E[c_2|c' < c_2 < c''] = \int_{c'}^{c''} c_2 dH(c_2),$$

the expected cost simplifies to

$$E[C(r)] = E[c_2] + pZ + p(X - Z) H(c'(r)) + \int_{c'(r)}^{c''(r)} (Z/r - pZ - c_2)\, dH(c_2)$$

Since H is strictly increasing and $c'(r)$ is strictly decreasing the term $p(X-Z)H(c'(r))$ strictly decreases in r. It is convenient to rewrite the last term as

$$E[C'(r)] = \int_{c'(r)}^{c''(r)} (Z/r - pZ - c_2)\, dH(c_2) = \int_{c'(r)}^{c''(r)} (c''(r) - c_2)\, dH(c_2).$$

From the definitions of $c'(r)$ and $c''(r)$ follows that $E\left[C'\left(\frac{1}{2}\right)\right] = 0$ and $E[C'(r)] > 0$ for any $r < \frac{1}{2}$. Therefore the expected cost is minimized at $r = \frac{1}{2}$ and this $\arg\min$ is unique. ■

Remark 1 *Note that the expected cost is strictly decreasing when r is reduced to $r = \frac{1}{2}$ from above. We are not arguing that the expected cost is monotonically decreasing when r is increased to $\frac{1}{2}$ from below, only that the minimum is reached only at $r = \frac{1}{2}$.*

Remark 2 *Intuitively the optimality of $r = 1/2$ follows from the fact that for given β and Z such r makes it most likely that both willing to finish and able to finish constraints are satisfied. The result is not trivial since, first, β itself depends on r and, second, β is a random variable at the time when r is chosen.*

Corollary 2 *With $X = Z$ any $r \geq 1/2$ is optimal.*

Note that with the optimal choice of r the expected cost of the project is $E[C] = E[c_2] + pZ + p(X-Z)H((2-p)Z)$ so that when X is high it may be optimal to introduce the award floor. Similarly to the case of unlimited budgets with $r \geq \frac{1}{2}$ it is optimal to choose $R = Z/q$ and set $r = \frac{1}{2}$, so that $c^* = (2-p)Z$ and $R = 2Z$. When $r < \frac{1}{2}$ the optimal award floor $R = Z/r$ and again it can be shown that optimal $r = \frac{1}{2}$. With optimal choice of r and $R = 2Z$ the expected cost of the project is

$$E[C]^* = 2ZH(c^*) + \int_{c^*}^{C} (c_2 + pZ)\, dH(c_2).$$

The same choice of $c^* = (2-p)Z$ is optimal without the award floor. The expected cost of the project is then

$$E[C]^{**} = \int_0^{c^*} (c_2 + pX)\, dH(c_2) + \int_{c^*}^{C} (c_2 + pZ)\, dH(c_2).$$

It is optimal to choose $r = \frac{1}{2}$ and install the award floor $R = 2Z$ if

$$E[c_2 | c_2 \leq (2-p)Z] > 2Z - pX.$$

5.3 Linear problems

Consider linear problem with $d_i = d$. We do not place restrictions on the sign of d, however we ignore the initial investment constraint

$$\beta \leq d/r,$$

and concentrate on the inequality

$$\beta \leq \max\left\{(Z+d)/r, Z/q\right\},$$

Proposition 3 *When $d_i = d > -Z$ for all bidders, optimal $r(d)$ solves $(Z+d)/r = Z/q$. When $d \leq -Z$, optimal $r = 0$ and the expected cost is strictly increasing in r.*

Proof. Note that for $d > Z$, $r(d) > 0$ and for all finite d, $r(d) < 1$. Consider linear problem with $d_i = d > Z$. Suppose in this problem it is optimal to set $r' > r(d)$. Then for every bidder only $(1-r')\beta \leq Z$ can be binding. As before introduce

$$c^*(r') = \frac{Z}{1-r'} - pZ \equiv \frac{Z}{q'} - pZ.$$

For bidder i the optimal bidding strategy satisfies

$$b^*(c_i) = \begin{cases} c_i/(1-pq') & \text{if } c_i < c^*(r') \\ c_i + pZ & \text{if } c_i \geq c^*(r') \end{cases}.$$

As before the project is finished by the winner if $c_2 \geq c^*(r')$ and costs $c_2 + pZ$ to the buyer. If $c_2 < c^*(r')$ the project is finished by the buyer and costs $c_2 + pX$. The expected cost is then increasing in c^* and therefore decreasing in r'. Starting from $r' > r(d)$ it is always optimal to reduce r'. The expected cost is therefore increasing in r when $r > r(d)$.

Suppose now that it is optimal to set $r' < r(d)$. Then only $r'\beta \leq Z + d$ can be binding. Given r' introduce

$$\begin{aligned} c'(r') &= (Z+d)/r' - p(Z+d)q'/r' = (Z+d)(1-p)/r' + p(Z+d) \\ \text{and } c''(r') &= (Z+d)/r' - pZ. \end{aligned}$$

Note that both $c'(r')$ and $c''(r')$ decrease with r'. Note that $c''(r(d)) = c'(r(d))$ and $c''(r') > c'(r')$ for $r' < r(d)$.

$$c''(r') - c'(r') = p(Z+d)/r' - p(Z+d) - pZ = p\left((Z+d)\frac{q'}{r'} - Z\right).$$

The optimal bidding strategy satisfies

$$b^*(c_i) = \begin{cases} c_i/(1-pq') & \text{if } c_i \leq c'(r') \\ (Z+d)/r' & \text{if } c''(r') > c_i > c'(r') \\ c_i + pZ & \text{if } c_i \geq c''(r'). \end{cases}$$

As in the case of $d = 0$, the expected cost of the project can be written as

$$E[C(r)] = E[c_2] + pZ + p(X - Z)H(c'(r')) + \int_{c'(r')}^{c''(r')} \left(\frac{Z + d}{r'} - pZ - c_2 \right) dH(c_2)$$

Since H is strictly increasing and $c'(r')$ is strictly decreasing, $p(X - Z)H(c'(r'))$ strictly decreases in r'. Consider now

$$E[C'(r)] = \int_{c'(r')}^{c''(r')} \left(\frac{Z + d}{r'} - pZ - c_2 \right) dH(c_2) = \int_{c'(r')}^{c''(r')} (c''(r') - c_2)\, dH(c_2)$$

From the definitions of $c'(r')$ and $c''(r')$ follows that $E[C'(r(d))] = 0$ and $E[C'(r')] > 0$ for any $r' < r(d)$. Therefore the expected cost is minimized at $r = r(d)$ and this $\arg\min$ is unique. ■

Note that

$$r(d) = (Z + d) / (2Z + d) \tag{2}$$

with $r(0) = \frac{1}{2}$. Moreover $r(d)$ is strictly increasing and $r(-Z) = 0$. Note that with $d < -Z$ for every bidder only the willing to finish constraint may be binding so that with $d < -Z$ we are in the case of unlimited budgets.

6 Arbitrary Budgets

Now we relax the constraint $m_i = c_i$ and deal with arbitrary c_i and m_i. For each bidder i it is convenient to introduce the deficit $d_i = c_i - m_i$. We need to consider the following constraints

$$r\beta \geq d_i, \tag{3}$$

When award β satisfies this constraint bidder i will make the initial investment of c_i. If, in addition, β satisfies

$$r\beta - d_i \geq Z, \tag{4}$$

bidder i will be *able* to cover the cost overrun due to the shock Z and if, in addition, β satisfies

$$(1 - r)\beta = q\beta \geq Z \tag{5}$$

bidder i will be *willing* to cover the cost overrun. Therefore for given award level β and shock value Z bidder i will face either constraint (4) or (5) if

$$\beta \leq \max\{(Z + d_i)/r, Z/q\}$$

We will ignore constraint (3) for the moment and derive the equilibrium only taking into account the constraints (4) and (5). We will later argue that with competitive banking sector and m_i representing the credit limit that the winner can secure $m_i \geq c_i$ so that constraint (3) will not be binding. Introduce $D(r)$ that satisfies

$$(Z + D(r))/r = Z/q.$$

Note that

$$D(r) = \left(\tfrac{1}{q} - 2\right) Z = \frac{2r - 1}{1 - r} Z \tag{6}$$

is increasing in r and $D(r)$ is inverse function to $r(d)$ in (2). Note also that $D(r) < c^*(r) = \left(\tfrac{1}{q} - p\right) Z$. Note also that $d_i > D$ implies that $(Z + d_i)/r > Z/q$ for bidder i, that is only constraint (4) can be binding. Just like in the case of lean budgets for bidder i with cost c_i and deficit $d_i > D$ it is therefore optimal to bid according to

$$b^*(c_i, d_i) = \begin{cases} c_i/(1 - pq) & \text{if } c_i/(1 - pq) \leq (Z + d_i)/r \\ (Z + d_i)/r & \begin{array}{c}\text{if } c_i/(1 - pq) > (Z + d_i)/r \text{ and} \\ c_i + pZ < (Z + d_i)/r\end{array} \\ c_i + pZ & \text{if } c_i + pZ \geq (Z + d_i)/r. \end{cases} \tag{7}$$

Note that for bidder i with given d_i introduce $c_i''(d_i)$, the solution to $c_i''(d_i) + pZ = (Z + d_i)/r$ and $c_i'(d_i)$, the solution to $c_i'(d_i)/(1 - pq) = (Z + d_i)/r$. Note that for each bidder i, $d_i > D$ implies that $c_i''(d_i) > c_i'(d_i)$. Indeed,

$$c_i''(d_i) = \frac{Z + d_i}{r} - pZ > \frac{Z + d_i}{r} - p(Z + d_i)\frac{q}{r} = c_i'(d_i),$$

where the inequality follows from $d_i > D$.

For bidder i with $d_i \leq D$, only constraint (5) can be binding, the strategy therefore changes regime at $c^* = Z/q - pZ$. For bidder i with cost c_i and deficit $d_i \leq D$ it is optimal to bid according to

$$b^*(c_i) = \begin{cases} c_i/(1 - pq) & \text{if } c_i < c^* \\ c_i + pZ & \text{if } c_i \geq c^* \end{cases}. \tag{8}$$

Note that c^* solves $c^*/(1 - pq) = (Z + D)/r$. Therefore all the bidders with types $c_i \leq c^*$ are affected by either (4) or (5) and all bid $c_i/(1 - pq)$ irrespective of their budget s_i. For bidders with $c_i > c^*$ and $d_i \leq D$ neither of the constraints is binding and they bid $c_i + pZ$ again irrespective of their budget. For bidders with $d_i > D$ only constraint (4) can be binding. Budgets m affect the bidding strategies only through the thresholds at which the regime changes from bidding $c/(1 - pq)$ to bidding $c + pZ$. Note that for bidder i with c_i and d_i for the regime to change $c_i/(1 - pq) = (Z + d_i)/r$ must hold. Since $Z > 0$, $d_i = c_i - s_i$ and $r < 1 - pq$, for low enough s_i the change of regime will never occur. Bidders with $m_i = 0$, in particular, always bid according to $c_i/(1 - pq)$.

Consider bidders with $d_i > D$. For given $c > c^*$ introduce $m^-(c)$, the solution to

$$\frac{Z + c - m^-(c)}{r} = c/(1 - pq).$$

Note that $m^-(c^*) = (2-p)Z$ so that for type $(c^*, m^-(c^*))$ the corresponding $d = D$. Therefore ray $(c, m^-(c))$ originates at $(c^*, (2-p)Z)$ on the line $d_i = D$. Since $r < 1 - pq$ ray $(c, m^-(c))$ has positive slope $\frac{\partial m^-(c)}{\partial c} = 1 - r/(1-pq)$. Given c all the bidders with deficits $d_i > D$ and budgets $m_i \leq m^-(c)$ bid according to $c/(1-pq)$.

Again consider bidders with $d_i > D$ and for given $c > c^*$ introduce $m^+(c)$, the solution to

$$\frac{Z + c - m^+(c)}{r} = c + pZ.$$

Again $m^+(c^*) = (2-p)Z$ so that $c^* - m^+(c^*) = D$. The slope of ray $(c, m^+(c))$ is $\frac{\partial m^+(c)}{\partial c} = 1 - r > \frac{\partial m^-(c)}{\partial c}$. Therefore the rays $(c, m^+(c))$ and $(c, m^-(c))$ have the same origin and $(c, m^+(c))$ always lies above $(c, m^-(c))$. Given c all the bidders with deficits $d_i > D$ and budgets $m_i \geq m^+(c)$ bid according to $c + pZ$.

Bidders with $d_i > D$, $c_i > c^*$ and $m_i \in (m^-(c_i), m^+(c_i))$ such that $(Z + d_i)/r > c + pZ$ and $(Z + d_i)/r < c/(1-pq)$ bid $(Z + d_i)/r$. For these types of bidders budgets m_i affect the bids, however, only in a particular combination with c_i.

Figure 3 represents the isobid curves in the space (c, m) with $D < 0$, $r < \frac{1}{2}$. Solid lines join the types that submit the same bid in equilibrium.

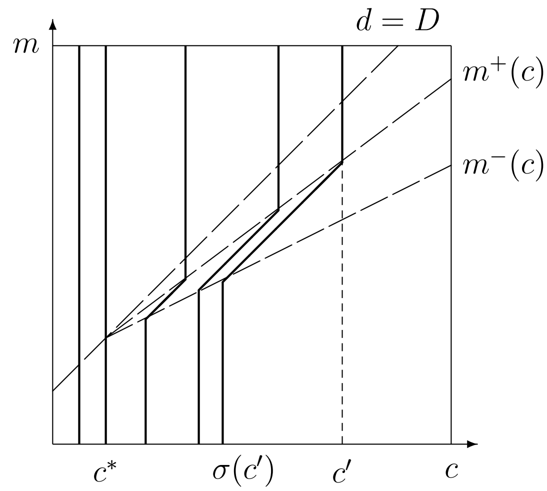

Figure 4: Isobid curves. Note the change of regimes for $c > c^*$

Note that for $c \leq c^*$ the isobid curves are vertical. To construct the isobid curve for a higher cost type fix $c' > c^*$, the cost of a type with $d_i < D$ that bids $c' + pZ$ in equilibrium. The corresponding type that submits the *same* bid but bids according to $c/(1-pq)$ is $\sigma(c') = (1-pq)(c'+pZ) < c'$. For such bidder $d_i > D$. Note that

$$0 < \frac{\partial \sigma(c)}{\partial c} = 1 - pq < 1,$$

that is when c rises, $\sigma(c)$ rises at a lower rate. Therefore the types that submit bids b' and $b'' = b' + \delta$ for any $\delta > 0$ "lie denser" for low s where they bid according to $c/(1-pq)$ and lie less dense for high m where they bid according to $c + pZ$. Recall that $m^-(c)$ and $m^+(c)$ define the thresholds where the bidding changes the regime for bidder with cost c and deficit $d < D$. The isobid curve therefore is vertical for $m \geq m^+(c')$ and $m \leq m^-(c')$ and involves kinks at $(c', m^+(c'))$ and $(\sigma(c'), m^-(c'))$. In between the kinks the isobid curve slopes upwards along $d_i = Const$. Note also that

$$\begin{aligned} \frac{\partial m^+(c)}{\partial c} &= q < 1 \text{ and} \\ \frac{\partial m^-(c)}{\partial c} &= \frac{q - pq}{1 - pq} < 1 \end{aligned}$$

so that the isobid curves that correspond to different c do not intersect.

At this stage we would like to motivate an additional on the budget constraints. Suppose that budget m_i represents the credit limit. Our paper is concerned with the situations in which the winning bidder has a signed contract with a buyer, the government or private party that stipulates the award that the contractor receives at the start and upon completion of the project. This contract can be viewed as collateral. Note that in equilibrium in every realization, regardless of whether the project is finished or not the contractor receives at least c_i as a payment. Competitive banking sector will therefore lend at least c_i to the winner of the tender prior to the initial investment stage. In what follows we assume that $m_i \geq c_i$ for every bidder. Note that our assumption does not remove budget constraints entirely since the winner of the tender may run into a cost overrun. The banks are naturally reluctant to extend his credit line in such situations. What is guaranteed though and what motivates our assumption is that in any realization of the shock in equilibrium whether the winner of the tender finishes the project or decides to quit he has enough cash to pay back c_i. Note also that m_i is still to be understood as exogenously given budget of bidder i and constitutes her private information.[13]

Assumption *For every contractor* $m_i \geq c_i$.

With such additional assumption constraint (3) does not bind and our equilibrium construction is complete.

[13] Our intention is to abstain from explicitly modeling banking sector, since we realize that by introducing it formally we create avenues for off equilibrium behaviour. However, it would be unrealistic to say that the winner of the government procurement contract cannot borrow against the contracted award.

6.1 Expected Cost of the Project

The difficulty with calculating the expected cost with arbitrary budgets stems from the fact that the equilibrium just derived depends on the budgets and we are not guaranteed that the bid of the contractor with the second lowest cost determines the award. Indeed, it may be that the bidder with the second lowest cost has a large deficit but the bidder with, say, third lowest cost has smaller deficit and bids more aggressively. In fact, we are not even guaranteed that the winner is the contractor with the lowest private component of the cost.[14] We however, will make an argument for the general case through a sequence of arguments for the cases each of which deals with degenerate $F(c, m)$. Consider again a procurement problem where $d_i = d$ for each bidder i. This corresponds to the case where costs and budgets for each bidder are distributed on a line parallel to $d = D$. This problem has one dimensional private information and therefore the award will be determined by the bid of the contractor with the second lowest cost just as in the case of lean budgets, Section 5.1. As is easy to see that the equilibrium in this degenerate distribution problem will be either of the type studied in Section 4 for unlimited budgets and involve a kink or the one from Section 5.1 and $r < \frac{1}{2}$, that is, involve a step. Which of the two cases arises depends on whether $d \geq D$ or $d < D$ applies, that is whether the line $d_i = d$ crosses the area in between the rays $(c, m^-(c))$ and $(c, m^+(c))$ or lies wholly to the left of it, see Figure 4 with the isobid curves.

For $d < D$ straightforward replication of Proposition 1 suggests that it is optimal to choose $r = 0$. For $d \geq D$ equally straightforward replication of Proposition 2 suggests that it is optimal to choose $r = \frac{1}{2}$. Note that neither of the arguments depends on the distribution of c_2 and therefore the corresponding optimal r would remain optimal if the award were always determined by c_3 or $c_{5.}$. Now consider a hypothetical procurement problem $\{c_2\}$ where the private information is two dimensional but the award is always determined by the bid of the contractor with the second lowest cost. Note that in equilibrium the project will be completed by the winner when neither of the constraints bind. The cost of the project is then $c_2 + pZ$ for $c_2 \geq c^*$ and $m \geq m^+(c_2)$. The project is also completed by the winner when $c_2 \geq c^*$ and $m^-(c_2) \leq m < m^+(c_2)$ and costs the government $(Z + d_2)/r > c_2 + pZ$ in these realizations. In the realizations with $c < c^*$ the bidder with the second lowest cost bids $c_2/(1 - pq)$, the winner is unwilling to finish the project in case of a shock. The project is completed by the winner only with probability $1 - p$. The government pays only the ex-ante fraction of the award and finishes the project with probability p. The expected cost in these realizations is $c_2 + pX$. With $c > c^*$ and $m < m^-(c_2)$ the winner is willing to finish the project but is unable to because of the large deficit. The cost to the government is again $c_2 + pX$. Figure 5 places the expected payments in the respective areas when $r < \frac{1}{2}$.

[14]Same applies to open descending price tender and results from the fact that the budget constraints are present in the model, not from our choice of the tender mechanism.

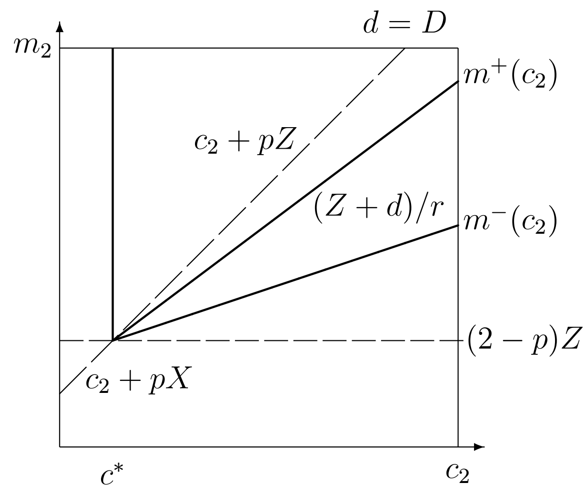

Figure 5: Expected cost of the project by realization

With $d_i \geq 0$ only the upper part of the expected costs diagram matters. Assume $r \geq \frac{1}{2}$, then $D \geq 0$ and $d = D$ lies to the left of $d = 0$. Figure 6 represents the expected payments for this case.

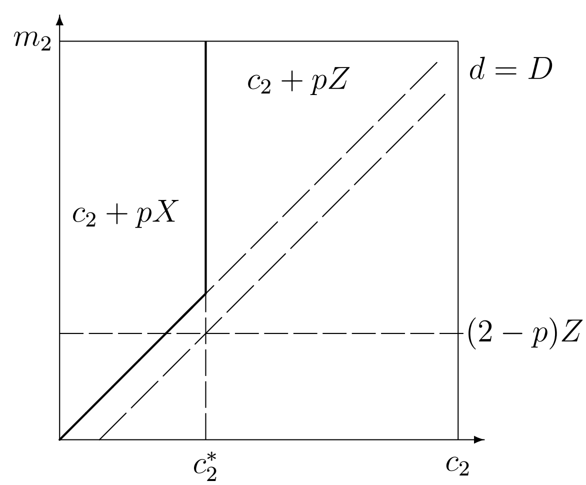

Figure 6: Expected cost of the project by realization $r \geq \frac{1}{2}$

It is obvious from the picture that with $r \geq \frac{1}{2}$ and $X > Z$ it is optimal to minimize c^* that is to choose $r = \frac{1}{2}$. Now consider $r < \frac{1}{2}$. We can further show that the optimal $r \in \left(0, \frac{1}{2}\right)$. Note that on each of the lines $d = Const$ the optimal r is either 0 or $\frac{1}{2}$. Note also that the calculation of the expected cost of the project can be performed as integration of the costs over the area above $d = 0$, since $m_i \geq c_i$ for every i. This integration in our hypothetical case, where c_2 always determines the award, can be performed in a conventional way, first integrate over c_2 and for each c_2 over m_2 or in the system of coordinates where the outer integral is over a variable orthogonal to $d = 0$ and the inner integral along the lines parallel to $d = 0$. Such integration amounts to averaging out the cases where $r = 0$ is optimal with the cases where $r = \frac{1}{2}$ is optimal. Therefore optimal r in the hypothetical procurement problem satisfies $0 < r < \frac{1}{2}$.

Now consider another hypothetical procurement problem $\{c_3\}$ where the award is always determined by the bid of the bidder with the third lowest cost. Since none of the arguments above explicitly depends on the distribution again optimal r satisfies $0 < r < \frac{1}{2}$. Note that in our general procurement problem there are realizations where the award is determined by c_2, realizations where it is determined by c_3, etc. The optimal r in the general procurement problem is again the weighted average of the optimal r's in hypothetical procurement problems.

Proposition 4 *With $X > Z$ optimal r satisfies* $0 < r < \frac{1}{2}$.

Corollary 3 *With $X = Z$ optimal* $r \geq \frac{1}{2}$.

Note that with $X > Z$ fully ex-post payment is optimal with unlimited budgets and suboptimal when budget constraints may bind. The result for the Corollary follows from the fact that $r \geq \frac{1}{2}$ is optimal for every $d \leq D$ and any r is optimal for $d > D$. Note that optimal r for $X > Z$ is not a subset of the set of optimal r for $X = Z$. Indeed with $X = Z$ the bankruptcy problem is absent, the buyer can finish the project at the same cost as the contractor. Therefore it is optimal to motivate aggressive bidding for the right to invest initial c_i. The award paid fully ex-ante does precisely that. With $X > Z$ the buyer has to balance the incentives to complete the project with the ability to complete it. Intermediate r is more suitable for the task.

7 Arbitrary Shocks

In this section we generalize our findings to the case where shock z is drawn from distribution with continuous c.d.f. F_z on support $[0, Z^{\max}]$[15] Apart from being more general such modification of the shocks process supports other aspects of the model. With the shock on two possible values 0, and Z the fact that the original contractor walked away from the project reveals to all the parties that the shock is indeed Z. Therefore it can be argued that any contractor that replaces the original one should not be paid more than Z, however, the assumption that $X > Z$ is crucial for the results. In this section all that can be possibly revealed by the original contractor quitting the project is that $z \geq \overline{z}$ where $\overline{z}$ is some endogenously determined threshold. In the spirit of our original model we do not model the process of tendering or contracting to fix the cost overrun, but rather assume that this process generates

[15] The distribution F_z studied before was placing mass p on Z and the remainder on 0.

price $z+\delta$ that the buyer is liable to pay for the cost overrun. The value of $\delta > 0$ is exogenously given.[16] Again we stress that $z+\delta$ is being paid *not* to the original contractor who is given *no* indication that his fixed price contract can be turned into cost plus contract in case of cost overruns.

We derive the equilibria and optimal r for the two leading examples, unlimited and lean budgets. We do not get closed form solutions and instead reason from the equations that describe the bidding strategies. Apart from introducing more generality in the model this modification of the shock process allows for more smooth bidding strategies, for given r, the bidding strategy is not separated into regions anymore and make the assumption that the ex-post shock is not verifiable more plausible.

7.1 Unlimited budgets

Given the award β the project is finished if z is such that

$$z \leq (1-r)\,\beta.$$

Then bidder i with cost c_i who bids b when the lowest bid submitted by the opponents is β_1 receives the payoff[17]

$$\pi_i\left(c_i, b, \beta_1\right) = \left(\beta_1 - c_i\right) F_z\left[(1-r)\,\beta_1\right] - \int_0^{(1-r)\beta_1} z dF_z + \left(r\beta_1 - c_i\right)\left(1 - F_z\left[(1-r)\,\beta_1\right]\right).$$

This results in bidder i's expected payoff

$$E\left[\pi_i\left(c_i, b\right)\right] = \int_b^{\beta^{-1}(\bar{c})} \left(r\beta_1 + \int_0^{(1-r)\beta_1} \left[(1-r)\,\beta_1 - z\right] dF_z - c_i\right) dG\left(\beta_1\right),$$

where G is the distribution of β_1. Consider

$$W\left(\beta_1, r\right) = r\beta_1 + \int_0^{(1-r)\beta_1} \left[(1-r)\,\beta_1 - z\right] dF_z.$$

Such $W\left(\beta_1, r\right)$ is increasing in β_1 and $W(0, r) = 0$. Optimal $b_W\left(c_i, r\right)$ then solves

$$W\left(b_W, r\right) = c_i. \tag{9}$$

[16] The buyer here acts as a price taker. Of course, when the replacement contractor asks for $z+\delta$ the buyer learns the true value of the shock and may be strategic but all we need for the result is that $\delta > 0$ so for small δ we believe it is reasonable to assume that the buyer acts as a price taker at this stage.

[17] This and similar expressions remain valid for $(1-r)\,\beta_1 > Z^{\max}$ with the convention that $dF_z = 0$ for $z < 0$ and $z > Z^{\max}$. This implies in particular that $\int_0^Y z dF_z = E[z]$ and $\int_Y^{Z^{\max}} dF_z = 0$ if $Y > Z^{\max}$.

Note that $b_W(c_i, r)$ that satisfies the above increases in c_i for given r and decreases in r for given c_i. Note also that with $F_z[(1-r)b] = 1$ for any b, that is in the model with no ex-post shocks, optimal $b_i = c_i$ for any r. It can be verified that $(1-r)\,b_W(c_i, r)$ strictly decreases in r for given c_i. Note also that

$$(1-r)\,bF_z[(1-r)\,b] + rb = W(b, r) - \int_0^{(1-r)b} z dF_z. \tag{10}$$

The buyer's expected cost depends on δ and distribution of z and c_2. The project is completed if $z \leq (1-r)\,b_2$, b_2 is then paid to the contractor, and not completed otherwise with the total contracted payment and the payment for completion $rb_2 + z + \delta$. The exact value of z is not known at this stage, all that the buyer knows is that $z > (1-r)\,\beta$, where award β is itself endogenously determined by the second lowest bid. This results in the expected cost of the project

$$E[C] = \int_{C^{\min}}^{C^{\max}} \left[b_2 F_z[(1-r)\,b_2] + rb_2(1 - F_z[(1-r)\,b_2]) + \int_{(1-r)b_2}^{Z^{\max}} (z+\delta)\, dF_z \right] dH,$$

where H stands for the distribution of the second lowest cost. The expected cost can be simplified to form

$$E[C] = \int_{C^{\min}}^{C^{\max}} \left[(1-r)\,b_2 F_z[(1-r)\,b_2] + rb_2 + \int_{(1-r)b_2}^{Z^{\max}} (z+\delta)\, dF_z \right] dH$$

Note from equations (9) and (10) that the first two terms form $c_2 + \int_0^{(1-r)b} z dF_z$ therefore the expected cost can be reduced to

$$E[C] = E[c_2] + E[z] + \delta \int_{C^{\min}}^{C^{\max}} \int_{(1-r)b_W(c_2,r)}^{Z^{\max}} dF_z dH(c_2) \tag{11}$$

Since $(1-r)\,b_W(c_2, r)$ strictly decreases in r for every c_2, the expected cost is minimized by setting $r = 0$.[18]

7.2 Lean Budgets

When $d_i = 0$ the project will be completed if

$$z \leq \min\{r, (1-r)\}\,\beta,$$

where again β is the award which is endogenously determined.

[18]Note that for some c, it may be that $(1-r)\,b_W(c,r) > Z^{\max}$ for some $r > 0$ so that further decrease in r does not decrease the value of the inner integral. Since $b_W(0,r) = 0$, for sufficiently small $c > C^{\min}$, $(1-r)\,b_W(c,r) < Z^{\max}$ as $Z^{\max} > C^{\min}$ so that the expected cost is strictly increasing in r.

Case 1 $r \leq \frac{1}{2}$.

When $r \leq \frac{1}{2}$ then only the constraint $z \leq r\beta$ can be binding and the payoff of bidder i with cost c_i who bids b

$$\pi_i(c_i, b, \beta_1) = (\beta_1 - c_i) F_z[r\beta_1] - \int_0^{r\beta_1} z dF_z + (r\beta_1 - c_i)(1 - F_z[r\beta_1])$$

After collecting the terms the expected payoff

$$E[\pi_i(c_i, b)] = \int_b^{\beta^{-1}(\bar{c})} \left[r\beta_1 + \int_0^{r\beta_1} [(1-r)\beta_1 - z] dF_z - c_i \right] dG(\beta_1)$$

Note that for $r \leq \frac{1}{2}$ the integral with respect to F_z is positive. Consider

$$\begin{aligned} A(\beta_1, r) &= r\beta_1 + \int_0^{r\beta_1} [(1-r)\beta_1 - z] dF_z. \\ W(\beta_1, r) &= r\beta_1 + \int_0^{(1-r)\beta_1} [(1-r)\beta_1 - z] dF_z. \end{aligned}$$

$A(\beta_1, r)$ is strictly increasing in β_1 and $A(0, r) = 0$. Therefore the expected payoff for bidder i is maximized when $A(b, r) = c_i$. For $r \leq \frac{1}{2}$ for bidder i it is optimal to submit bid $b_A(c_i, r)$ that solves

$$A(b_A, r) = c_i \tag{12}$$

Note that $b_A(c_i, r)$ is increasing in c_i for given r and decreases in r for given c_i. Note also that in the model with no ex-post shocks $F_z[rb] = 1$ and it is optimal to bid $b_i = c_i$. Note also that $A\left(b, \frac{1}{2}\right) = W\left(b, \frac{1}{2}\right)$ so that at $r = \frac{1}{2}$ equations (12) and (**??**) are identical. The expected cost of the project

$$E[C] = \int_{C^{\min}}^{C^{\max}} \left[b_2 F_z[rb_2] + rb_2(1 - F_z[rb_2]) + \int_{rb_2}^{Z^{\max}} (z + \delta) dF_z \right] dH_2$$

Equivalently

$$E[C] = \int_{C^{\min}}^{C^{\max}} \left[rb_2 + (1-r) b_2 F_z[rb_2] + \int_{rb_2}^{Z^{\max}} (z + \delta) dF_z \right] dH_2$$

which using (12) can be simplified to.

$$E[C] = E[c_2] + E[z] + \delta \int_{C^{\min}}^{C^{\max}} \int_{rb_A(c_2, r)}^{Z^{\max}} dF_z dH(c_2)$$

Again direct computation shows that $rb_A(c_2, r)$, is strictly increasing in r for every c_2. Therefore with $\delta > 0$ and $r \leq \frac{1}{2}$ optimal $r = \frac{1}{2}$ and the expected cost is strictly decreasing in r when $r < \frac{1}{2}$.

Case 2 $r \geq \frac{1}{2}$.

With $r \geq \frac{1}{2}$ as before only constraint $z \leq (1-r)\beta$ can bind and for bidder i with cost c_i it is optimal to submit $b(c_i, r)$ that solves (**??**) above. The expected cost of the project is given by (11) and is minimized by $r = \frac{1}{2}$. In addition the expected cost is strictly increasing in r when $r > \frac{1}{2}$. Therefore with lean budgets optimal $r = \frac{1}{2}$. We summarize the results form this Section so far in

Proposition 5 *With arbitrary distribution of shocks and unlimited budgets $r = 0$ is optimal, with lean budgets $r = \frac{1}{2}$ is optimal.*

7.3 Linear problems

Again we ignore the initial investment phase and concentrate on the reinvestment. After initial investment of c_i bidder i with deficit d will complete the project if

$$z \leq \min\{r\beta - d, (1-r)\beta\},$$

Clearly when d is low enough only the willing to finish constraint may be binding and optimal $r = 0$. For d that are not so low there exists optimal $r(d) \in (0,1)$ such that if $r > r(d)$, for every bidder only the willing to finish constraint $z \leq (1-r)\beta$ may be binding, and if $r < r(d)$, only the able to finish constraint $z \leq r\beta - d$ may be binding.

Suppose $r > r(d)$ then by definition of $r(d)$ bidder i with cost c_i bids according to $b_W(c_i, r)$ that solves

$$W(b_W, r) = c_i, \text{ with } W(b, r) = rb + \int_0^{(1-r)b} [(1-r)b - z]\, dF_z, \tag{13}$$

as in the case of unlimited budgets. Recall that $(1-r)b_W(c_2, r)$ strictly decreases in r for every c_2 and the expected cost

$$E[C] = E[c_2] + E[z] + \delta \int_{C^{\min}}^{C^{\max}} \int_{(1-r)b_W(c_2,r)}^{Z^{\max}} dF_z dH(c_2)$$

is strictly increasing when $r > r(d)$.

Now consider the case where $r < r(d)$. Then only the constraint $z \leq r\beta - d$ can be binding and the payoff of bidder i with cost c_i who bids b

$$\pi_i(c_i, b, \beta_1) = (\beta_1 - c_i) F_z[r\beta_1 - d] - \int_0^{r\beta_1 - d} z dF_z + (r\beta_1 - c_i)(1 - F_z[r\beta_1 - d])$$

After collecting the terms the expected payoff

$$E[\pi_i(c_i, b)] = \int_b^{\beta^{-1}(\bar{c})} \left[r\beta_1 + \int_0^{r\beta_1 - d} [(1-r)\beta_1 - z]\, dF_z - c_i \right] dG(\beta_1)$$

Note that for $r < r(d)$, $r\beta_1 - d < (1-r)\beta_1$ so that the integral with respect to F_z is positive. Consider

$$\begin{aligned} A(\beta_1, d, r) &= r\beta_1 + \int_0^{r\beta_1 - d} [(1-r)\beta_1 - z]\, dF_z. \\ W(\beta_1, r) &= r\beta_1 + \int_0^{(1-r)\beta_1} [(1-r)\beta_1 - z]\, dF_z. \end{aligned}$$

It is strictly increasing in β_1 and $A(0,d,r) = -\int_0^{-d} z dF_z \leq 0$. If $d > 0$ then $A(0,d,r) = 0$ since the support of z is to the right of 0. Therefore the expected payoff for bidder i is maximized when $A(b,d,r) = c_i$. For $r \leq r(d)$ for bidder i it is optimal to submit bid $b_A(c_i, d, r)$ that solves

$$A(b_A, d, r) = c_i, \text{ with } A(b,d,r) = rb + \int_0^{rb-d} [(1-r)b - z]\, dF_z \qquad (14)$$

Note that $b_A(c_i, d, r)$ is increasing in c_i for given r and d, increases in d for given c_i and r and decreases in r for given c_i and d. Note also that in the model with no ex-post shocks $F_z[rb - d] = 1$ and it is optimal to bid $b_i = c_i$. Note also that $A(b, d, r(d)) = W(b, r(d))$ so that at $r = r(d)$ equations (14) and (13) are identical. The expected cost of the project when $r < r(d)$

$$E[C] = \int_{C^{\min}}^{C^{\max}} \left[b_2 F_z[rb_2 - d] + rb_2(1 - F_z[rb_2 - d]) + \int_{rb_2 - d}^{Z^{\max}} (z+\delta)\, dF_z \right] dH_2$$

Equivalently

$$E[C] = \int_{C^{\min}}^{C^{\max}} \left[rb_2 + (1-r) b_2 F_z[rb_2 - d] + \int_{rb_2 - d}^{Z^{\max}} (z+\delta)\, dF_z \right] dH_2$$

which using (14) can be simplified

$$E[C] = E[c_2] + E[z] + \delta \int_{C^{\min}}^{C^{\max}} \int_{rb_A(c_2,d,r)-d}^{Z^{\max}} dF_z dH(c_2)$$

Again direct computation shows that $rb_A(c_2, d, r) - d$ is strictly increasing in r for every c_2 and d. Therefore with $\delta > 0$ and $r < r(d)$ the expected cost is strictly decreasing in r. Optimal $r(d)$ therefore solves

$$\int_{C^{\min}}^{C^{\max}} \int_{(1-r(d))b_W(c_2, r(d))}^{Z^{\max}} dF_z dH_2 = \int_{C^{\min}}^{C^{\max}} \int_{r(d) b_A(c_2, d, r(d)) - d}^{Z^{\max}} dF_z dH_2, \qquad (15)$$

where $b_W(c, r)$ solves (13) and $b_A(c, d, r)$ solves (14).

Note that if for given c_2, $(1 - r(d))\, b_W(c_2, r(d)) \geq Z^{\max}$, the corresponding inner integral in LHS is 0. However, since $C^{\min} < Z^{\max}$ and $b_W(0, r) = 0$, for small enough

c_2 the corresponding $(1-r(d))\, b_W(c_2, r(d)) < Z^{\max}$ so that LHS(r) in (15) is strictly positive for any r and attains its minimum at $r=0$, see footnote 17. In contrast, there are pairs r and d, for instance, $r=0$ and any $d < -Z^{\max}$ such that RHS(r,d) in (15) is zero, and more generally the pairs r and d such that RHS$(r,d) < \{\text{LHS}(0)\}$. For such d only the willing to finish constraint may be binding and therefore optimal $r=0$. For r and d such that (15) has a solution, the solution $r(d)$ is unique. Indeed LHS is strictly increasing in r and RHS is strictly decreasing in r for any d. Also since LHS strictly increases in r and is independent of d, while RHS is strictly decreasing in r and strictly increasing in d, the solution $r(d)$ is strictly increasing in d.

From $A\left(b, 0, \frac{1}{2}\right) = A\left(b, \frac{1}{2}\right) = W\left(b, \frac{1}{2}\right)$ follows that $r(0) = \frac{1}{2}$. Since RHS$(0, -Z^{\max}) <$ LHS(0) and RHS is strictly increasing in d there exists threshold $d^* > -Z^{\max}$ such that $r(d^*) = 0$ and $r(d) > 0$ for any $d > d^*$.

7.4 Arbitrary budgets

For bidders with $d_i \leq d^*$ only the willing to finish constraint may bind, therefore they bid according to $b_W(c_i, r)$. To determine the bidding strategies for those with $d > d^*$ given strictly increasing $r(d)$ introduce its inverse $D(r)$. This $D(r)$ plays exactly the same role as $D(r)$ given by (6) for the case where the value of the (only possible) shock Z is common knowledge. For given r for the bidders with $d_i > D(r)$ only the able to finish constraint can be binding, therefore they bid according to $b_A(c_i, d_i, r)$ such that

$$A(b_A, d_i, r) = c_i, \text{ with } A(b, d_i, r) = rb + \int_0^{rb-d_i} [(1-r)\, b - z]\, dF_z \tag{16}$$

whereas for the bidders with $d_i \leq D(r)$ only the willing to finish constraint can be binding, therefore they bid according to $b_W(c_i, r)$ such that

$$W(b_W, r) = c_i, \text{ with } W(b, r) = rb + \int_0^{(1-r)b} [(1-r)\, b - z]\, dF_z.$$

Note that even though the value of the future shock is not known to the bidders at the tender stage they have enough information to compute $r(d)$ and its inverse. Note that $d_i > D(r)$ guarantees that $rb - d_i < (1-r)\, b$ so that the integral in (16) is positive.

8 Conclusions

This paper concerns with the issue of cost overruns in public procurement projects which rightfully attracts the attention of the regulators. We view the problem of cost overruns as an incentive problem. Those problems in procurement literature were traditionally approached from two sides: contract theory provides incentives at the *after* the tender stage, auction theory provides incentives *at* the tender stage. The procurement game in this paper combines both at the tender and after the tender stages. We stand on the premise that when private information needs to be revealed competitive bidding, where *one* buyer deals *with multiple* contractors, provides better incentives than a clever contract in which the buyer is by definition locked in *one on one* relationship with the contractor. The leverage that the buyer has at the tender stage simply because she still has a choice of which contractor to deal with dissipates once the contract is signed if the contractor retains private information. The fixed price contract is therefore superior to cost plus contract since the terms are "negotiated" at the stage where the buyer still has the leverage and then stay unchanged by the definition of the contract. We therefore concentrate on fixed price contracts and offer the regulators a modest departure to the contract terms, the fixed award determined by the competitive bidding is to be split into the ex-ante and ex-post parts. We show that this approach has a potential to reduce the cost of the procured projects and lower the likelihood of the contractor's default.

Remark 3 *It may appear that the buyer is suffering from the consequences of the winner's curse by running an efficient procurement tender. Since the cost overrun is typically measured as the ratio of the actual cost to the original estimate* $(c+z)/c$ *and this is decreasing in* c*, the cost overruns are higher because the contractor with the low* c_i *is selected. This is not the reason to select the contractor with higher* c*, however. The cost overruns will be lower, but the actual cost, which is the one that the buyer is concerned about in this paper, will be higher.*

Remark 4 *Bankrupticies happen a lot according to Department of Fair Trading. <20'000 no home insurance. builders have up to 500'000 liabilities.*

References

[1] Ashley, D. and Workman A. "Incentives in Construction Contracts." Document SD-8, Austin, Tex.: The Construction Industry Institute, 1986.

[2] Arozamena, L. and E. Cantillion, (2004) "Investment Incentives in Procurement Auctions," *Review of Economic Studies*, 71, 1-18.

[3] Bajari, P. and S. Tadelis, (2001) "Incentives versus transaction costs: a theory of procurement contracts," *RAND Journal of Economics*, 32, 3, 387-407.

[4] Burguet R., J,-J. Ganuza and E, Hauk (2009): "Limited Liability and Mechanism Design in Procurement," mimeo.

[5] Che, Y. and Gale, I. (1998) "Standard Auctions with Budget Constrained Bidders," *Review of Economic Studies* 65, 1-21.

[6] Che, Y. and Gale, I. (2000) "The Optimal Mechanism for Selling to a Budget-Constrained Buyer", *Journal of Economic Theory* 92: 198-233.

[7] Crocker, K.J. and K.J. Reynolds (1993) "The Efficiency of Incomplete Contracts: An Empirical Analysis of Air Force Engine Procurement," *RAND Journal of Economic*s, 24, 126-146.

[8] Fox, J.R. Arming America: How the US Buys Arms. Cambridge: Harvard University Press, 1974.

[9] Laffont, J.J. and Tirole, J. A Theory of incentives in Procurement and Regulation. Cambridge. MIT Press, 1993.

[10] Manelli, A.M. and D.R. Vincent, (1995) "Optimal Procurement Mechanism", *Econometrica* 63(3): 591-620

[11] McAfee, P.R.. and J. McMillan, (1986). "Bidding for Contracts: A Principal-Agent Analysis," *RAND Journal of Economics*, Vol. 17, No. 3, pp. 326-338.

[12] ——— AND ———. Incentives in Government Contracting. Toronto: University of Toronto Press, 1988

[13] Myerson, R. B., (1981) "Optimal Auction Design," *Mathematics of Operations Research*, Vol. 6, No. 1, pp. 58-73.

[14] Pai, M.M. and Vohra, R. (2008) "Optimal Auctions with Financially Constrained Bidders," mimeo, Kellog School of Management, Northwestern University.

[15] Parlane, S. (2003) "Procurement Contracts under Limited Liability," *Economic and Social Review* 34(1): 1—21

[16] Peck, M.J. and F.M. Scherer, The weapons acquisition process: an economic analysis, Harvard University, 1962

[17] Piccione M. and G. Tan (1996), "Cost-Reducing Investment, Optimal Procurement and Implementation by Auctions," *International Economic Review*, 37, 3, 663-685.

[18] Riley, J. G. and W. F. Samuelson (1981), "Optimal Auctions," *American Economic Review*, 71, 3, 381-392.

[19] Riordan, M.H. and D. Sappington, (1988) "Commitment in Procurement Contracting," *Scandinavian Journal of Economics*, Vol. 90, No. 3, pp. 357-372.

[20] Tadelis, S. and P. Bajari (2006) "Incentives and Award Procedures: Competitive Tendering vs. Negotiations in Procurement," Handbook of Procurement, Dimitri, Pigo and Spagolo, Eds.

[21] Tirole, J., (1986), “Procurement and Renegotiation,” *Journal of Political Economy*, Vol. 94, No. 2, pp. 235-259